EECS 700: Exam # 1

Tuesday, October 21, 2014

Print Name and Signature____

The rules for this exam are as follows:

- **Write your name on the front page of the exam booklet. Initial each of the remaining pages in the upper-right hand corner. Sign the front of the exam booklet. Your exam will not be graded if you have not signed the front page of the booklet.**
- The exam has 5 questions and is 7 pages long (including this page). Be sure you have all of the pages before beginning the exam.
- This exam will last for 75 minutes.
- Show **ALL** work for partial/full credit. This includes any definitions, mathematics, figures, etc.
- The exam is closed book and closed notes.
- You are allowed to use a calculator on the exam provided you use it to perform arithmetic computations only and show all of your work on the exam.
- No collaboration of any kind is allowed on the exam.

1. ________ (15 pts)	4. ________ (15 pts)
2. ________ (15 pts)	5. ________ (15 pts)
3. ________ (15 pts)	T. ________ (75 points)

1. (15 points) Suppose that a quantity $x(t)$ changes in time according to the ODE

$$\frac{dx}{dt} = a + bx + cx^2.$$

(a) Show how to nondimensionalize this ODE.

(b) What is the advantage of nondimensionalization for this particular problem?

2. (15 points) Recall the Lotka-Volterra model which describes the predator-prey relationship of a simple ecosystem as follows:

$$\begin{aligned}\dot{x} &= ax - bxy \\ \dot{y} &= -cy + dxy,\end{aligned}$$

where $x(t)$ denotes the population of the prey species, and $y(t)$ denotes the population of the predator species and a, b, c, and d are assumed to be positive constants.

(a) Show how to extend the model to one which models the following closed ecosystem containing three species. Suppose that species x is a grass-grazer whose population in isolation would obey the logistic equation, and that it is preyed upon by species y who, in turn, is the sole food source of species z. Recall that the logistic equation is based on the introduction of a carrying capacity of the environment with respect to a particular species. **Be sure to define any new notation and clearly state any assumptions.**

(b) Give an example of such a three-species ecosystem.

3. (15 points) Consider the following model of love affairs dynamics proposed by Steven Strogatz:

$$
\begin{aligned}
\dot{x} &= \alpha_1 x(t) + \beta_1 y(t) \\
\dot{y} &= \beta_2 x(t) + \alpha_2 y(t),
\end{aligned}
$$

where $x(t)$ denotes Romeo's emotions (love if $x(t) > 0$, hate if $x(t) < 0$) for Juliet at time t, while $y(t)$ denotes Juliet's love/hate for Romeo at time t. The coefficients $\alpha_i, i = 1, 2$, reflect the influence of their own emotions on themselves, while β_i describe the direct effect of their love on their partner. Different signs of the coefficients $\alpha_i, \beta_i, i = 1, 2$, describe different romantic styles.

(a) Describe the various romantic styles in regards to Romeo's behavior as it relates to Juliet. Consider the following cases: $\alpha_1, \beta_1 > 0$, $\alpha_1 > 0, \beta_1 < 0$, $\alpha_1 < 0, \beta_1 > 0$, and $\alpha_1 < 0, \beta_1 < 0$.

(b) Compute and classify the equilibrium point(s) of the model for $\alpha_1 = -1, \beta_1 = 1, \alpha_2 = -1, \beta_2 = -1$. Interpret the meaning of each equilibrium point within the context of the application.

(c) Suppose the model is modified in order to incorporate the notion of appeal as follows:

$$
\begin{aligned}
\dot{x} &= \alpha_1 x(t) + \beta_1 y(t) + r_1 A_2 \\
\dot{y} &= \beta_2 x(t) + \alpha_2 y(t) + r_2 A_1,
\end{aligned}
$$

where A_1, A_2 are constant coefficients reflecting the appeal of Romeo and Juliet, respectively, r_1 describes Romeo's reaction to Juliet's appeal, and r_2 describes Juliet's reaction to Romeo's appeal.

How does the mathematical analysis of this model differ from that of the original model? How is this helpful from the perspective of modeling two individuals falling in love?

PLEASE WRITE YOUR ANSWER TO THIS QUESTION ON PAGE 5.

PLEASE WRITE YOUR ANSWER TO QUESTION 3 HERE.

4. (15 points) Consider the initial value problem

$$\begin{aligned} y' &= \lambda y, \\ y_0 &= y(0), \end{aligned}$$

and assume that $Re(\lambda) < 0$.

(a) Determine and draw the stability region for the backward Euler method as applied to this initial value problem.

(b) Would it be more appropriate to use the Forward Euler Method or the Backward Euler Method on an initial value problem containing a stiff ODE? Why? Consider stability, accuracy, and efficiency in your response.

5. (15 points) Consider the boundary value problem

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, 0 < x, y < 1$$

$$\mathrm{BC} : u(0, y) = 0; u(1, y) = 0; u(x, 0) = f(x); u(x, 1) = 0.$$

(a) Formulate a finite difference method for this boundary value problem. As part of the process, draw and label a computational grid/mesh that your finite difference method would use.

(b) Specify what calculations would need to be done in order to arrive at a numerical solution to the boundary value problem.