Academic Integrity Statement
“I have neither given nor received assistance on this exam.”

Print Name____________________________________
Signature____________________________________

The rules for this exam are as follows:

• Write your name on the front page of the exam booklet. Initial each of the remaining pages in the upper-right hand corner. Sign the academic integrity statement on the front of the exam booklet. Your exam will not be graded if you have not signed the front page of the booklet.

• The exam has 5 questions and 1 extra credit question. The exam is 7 pages long (including this page). Be sure you have all of the pages before beginning the exam.

• This exam will last for 75 minutes.

• Show ALL work for partial/full credit. This includes any definitions, mathematics, figures, etc.

• The exam is closed book and closed notes.

• Calculators and cell phones are not allowed on the exam. Cell phones can only be used for keeping track of time.

• No collaboration of any kind is allowed on the exam.

1. ______ (20 points)  5. ______ (10 points)
2. ______ (15 points)  EC. ______ (10 points)
3. ______ (15 points)  T. ______ (80 points)
4. ______ (20 points)
1. (20 points; 2 points each) Write **TRUE** or **FALSE** to the **LEFT** of each question. Do **NOT** include any additional justification.

(a) A task dependency graph can be used to determine the maximum amount of concurrency that is possible in a parallel program.

(b) The 15-puzzle problem is an example of a problem which employs speculative decomposition.

(c) The serial cyclic reduction algorithm is an efficient scheme for solving linear systems of equations involving a tridiagonal matrix.

(d) Reordering of the ijk loops in the parallel Gaussian Elimination algorithm leads to parallel algorithms with different performance when the partition of matrix $A$ is fixed.

(e) Use of a block-cyclic distribution in a parallel LU factorization code results in reduced idle time when compared with that of a block distribution.

(f) Reordering the computations in a parallel linear solver results in a different solution to the linear system.

(g) MPI is a parallel programming language.

(h) The use of MPI_Send and MPI_Recv in a parallel program does not result in a deadlock.

(i) MPI_Isend and MPI_Irecv are non-blocking communication operations.

(j) EECS 739 is a course on parallel scientific computing.
2. (15 points; 3 points each) Consider the task graph given below. Determine the following properties and give a brief justification for each.

(a) The maximum degree of concurrency.

(b) The critical path length.

(c) Maximum achievable speedup over one process assuming that an arbitrarily large number of processors is available.

(d) The minimum number of processors needed to obtain the maximum possible speedup.

(e) The maximum achievable speedup if the number of processors is limited to 4.
3. (15 points) Consider solving $Ax = b$, where $A = n \times n$ and $b = n \times 1$ are given as input. Next examine the following output obtained by running code for one of the linear solvers we studied in class on linear systems of various sizes.

<table>
<thead>
<tr>
<th>Matrix size</th>
<th>Flops computed</th>
</tr>
</thead>
<tbody>
<tr>
<td>$500 \times 500$</td>
<td>$2.25 \times 10^5$</td>
</tr>
<tr>
<td>$1000 \times 1000$</td>
<td>$1.8 \times 10^9$</td>
</tr>
<tr>
<td>$2000 \times 2000$</td>
<td>$1.44 \times 10^{10}$</td>
</tr>
</tbody>
</table>

(a) (10 points) Which of the following codes, i.e., $LDL^T$ Factorization, $LU$ Factorization, Backward Substitution, or Cyclic Reduction, was used to produce the above output? Justify your answer. **Note: It is sufficient to work with just the leading term in the complexity analysis.**

(b) (5 points) Suppose that the linear solver took $n$ milliseconds to solve the linear system for the $500 \times 500$ matrix. How many milliseconds would the linear solver take to solve a linear system with a $4000 \times 4000$ matrix?
4. (20 points) Define a checkerboard matrix to be an $n \times n$ matrix $A$ which is divided into $m^2$ equally-sized blocks (where $cm = n$) with nonzeros in the odd-numbered blocks and zeros in the even-numbered blocks. For this problem, consider checkerboard matrices with $n = 9$ and $m = 3$ as shown in the following figure.

Now consider multiplication of two checkerboard matrices $A$ and $B$.

(a) (5 points) Draw a diagram illustrating the structure of $A$, $B$, and $C$, where $C = A \times B$.

(b) (15 points) Suppose $p = (m^2 + 1)/2 = 5$ processors are available. Write parallel pseudocode for matrix-matrix multiplication of two checkerboard matrices. Label your diagram from (a) to indicate assignments to processors.
5. (10 points) Suppose that MPI_COMM_WORLD consists of the three processors 0, 1, and 2, and suppose the following code is executed in a distributed memory environment:

```c
int x, y, z;
switch(my_rank) {
    case 0: x=0; y=1; z=2;
        MPI_Bcast(&x, 1, MPI_INT, 0, MPI_COMM_WORLD);
        MPI_Send(&y, 1, MPI_INT, 2, 43, MPI_COMM_WORLD);
        MPI_Bcast(&z, 1, MPI_INT, 1, MPI_COMM_WORLD);
        break;
    case 1: x=3; y=8; z=5;
        MPI_Bcast(&x, 1, MPI_INT, 0, MPI_COMM_WORLD);
        MPI_Bcast(&y, 1, MPI_INT, 1, MPI_COMM_WORLD);
        break;
    case 2: x=6; y=7; z=8;
        MPI_Bcast(&z, 1, MPI_INT, 0, MPI_COMM_WORLD);
        MPI_Recv(&x, 1, MPI_INT, 0, 43, MPI_COMM_WORLD, &status);
        MPI_Bcast(&y, 1, MPI_INT, 1, MPI_COMM_WORLD);
        break;
}
```

What are the values of x, y, and z on each processor after the code has executed? Explain your answer.
1. (10 points) Write a pseudocode for parallel matrix-vector multiplication. In particular, let $M$ be an $n \times n$ matrix and $x$ be an $n \times 1$ vector. Your pseudocode should specify how to compute $v = M \cdot x$ for the case when $n = cp$, i.e., when $n$ is a multiple of $p$. Label each step in your pseudocode with the corresponding MPI function that would be used to perform the step.