The rules for this exam are as follows:

- **Write your name on the front page of the exam booklet.** Initial each of the remaining pages in the upper-right hand corner. Sign the front of the exam booklet. Your exam will not be graded if you have not signed the front page of the booklet.

- This exam will last for 50 minutes.

- Show **ALL** work for partial/full credit. This includes any definitions, mathematics, figures, etc.

- The exam is closed book and closed notes.

- A calculator is allowed provided it is only used to perform basic calculations. It cannot be programmed with algorithms or notes.

- No laptops, ipads, or other types of non-medical electronic devices are allowed.

- No collaboration of any kind is allowed on the exam.

1. ______ (15 points)  
2. ______ (15 points)  
3. ______ (15 points)  
4. ______ (15 points)  
5. ______ (20 points)  
EC. ______ (10 points)  
T. ______ (80 points)
1. (15 points; 3 points each) Please write TRUE or FALSE below each question. No additional justification is needed.

(a) Cancellation error occurs when subtracting two numbers of the same sign and similar magnitude.

(b) The two smallest positive numbers in a given floating-point system are underflow and machine epsilon.

(c) There are $2(\beta - 1)\beta^{p-1}(U - L + 1) + 1$ normalized floating point numbers in a given floating-point system.

(d) If the residual of a linear system is small, then the forward error is small.

(e) A symmetric positive definite matrix is always well-conditioned.
2. (15 points)

(a) (8 points) Which of the following two formulas is a better way of computing \( x^6 - 6x^5 + 15x^4 - 20x^3 + 15x^2 - 6x + 1 \) and why? Consider both efficiency and accuracy.

Option 1: \((((((x-6)x+15)x-20)x+15)x-6)x+1\)

Option 2: \(x^6 - 6x^5 + 15x^4 - 20x^3 + 15x^2 - 6x + 1\).

(b) (7 points) Explain why an alternating infinite series such as

\[ e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots \]

for \( x < 0 \) is difficult to evaluate accurately in floating point arithmetic.
3. (15 points) Compute the permutated $LU$ factorization of the following matrix using Gaussian elimination with partial pivoting:

$$A = \begin{pmatrix} 0 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}.$$ 

Write your answer in the form $PA = LU$. 


4. (15 points) Consider solving the linear system given by $Ax = b$. Given there is some uncertainty in $b$, in reality, you end up solving $A\hat{x} = b + \Delta b$.

How does this perturbation in $b$ affect the solution?

**Hint:** You will want to prove the following result:

$$\frac{\| \Delta x \|}{\| x \|} \leq \text{cond}(A) \frac{\| \Delta b \|}{\| b \|}.$$
5. (20 points; 10 points each) The following Matlab fragment computes the product \( C \) of an \( n \times n \) matrix \( A \) with \( n \times n \) upper triangular matrix \( U \).

(a) (10 points) Determine accurate to the leading term how many floating-point operations are used by this fragment. (Hint: Note that both multiplication and addition operations are occurring since * is operating on vectors. For example, \( C(1,2) = A(1,1:2) * U(1:2,2) \) which is \( A(1,1) * U(1,2) + A(1,2) * U(2,2) \).)

```matlab
for i = 1 : n
    for j = 1 : n
        C(i,j) = A(i,1:j) * U(1:j,j);
    end
end
```

(b) (10 pts.) Rewrite the fragment in the preceding question with better vectorization. In particular, use only one loop. **But be sure not to increase the number of flops required.**
**OPTIONAL: Extra-Credit Question** (10 points)

Complete the following Matlab function (or write the corresponding pseudocode) so that it performs as described. **Efficiency matters.**

```matlab
function [y,z] = SolveMe(A,C,g,h)

% % Input:
% A and C are n-by-n matrices with A nonsingular.
% g and h are column vectors of length n.
% %
% % Output:
% y and z are column vectors of length n satisfying:
% A*y+C*z = g
% A*z = h
```