EECS 639: Exam # 1
Wednesday, September 21, 2016

Print Name and Signature______________________________

The rules for this exam are as follows:

- Write your name on the front page of the exam booklet. Initial each of the remaining pages in the upper-right hand corner. Sign the front of the exam booklet. Your exam will not be graded if you have not signed the front page of the booklet.

- This exam will last for 50 minutes.

- Show ALL work for partial/full credit. This includes any definitions, mathematics, figures, etc.

- The exam is closed book and closed notes.

- A calculator is allowed provided it is only used to perform calculations manually.

- No laptops, ipads, or other types of non-medical electronic devices are allowed.

- No collaboration of any kind is allowed on the exam.

1. ______ (10 points) 5. ______ (10 points)
2. ______ ( 5 points) 6. ______ (15 points)
3. ______ ( 5 points) EC. ______ (10 points)
4. ______ (10 points) T. ______ (55 points)
1. (10 points; 2 points each) Please write TRUE or FALSE below each question. No additional justification is needed.

(a) \( \pi \) is a machine representable number in a base-2 floating point number system.

(b) Catastrophic cancellation is the type of error that is exhibited by computing

\[
\]

(c) The following matrix is ill-conditioned:

\[
A = \begin{pmatrix}
10^{-10} & 0 \\
0 & 10^{50}
\end{pmatrix}.
\]

(d) Gaussian Elimination can be used to reduce the following matrix into an upper triangular matrix:

\[
A = \begin{pmatrix}
3 & 2 & 1 \\
6 & 4 & 2 \\
0 & 1 & 0
\end{pmatrix}.
\]

(e) If the residual of a linear system is small, then the forward error in the solution is small.
2. (5 points) Consider the following computation being performed on a floating point number system having an underflow level of UFL = 10^{-38}: \( u = (v \times w)/(y \times z) \), with \( v = 10^{-15}, w = 10^{-30}, y = 10^{-20}, \) and \( z = 10^{-25} \). Clearly indicate which (if any) parts of this computation will underflow. In each case where underflow occurs, is it reasonable simply to set to zero the quantity that underflows?

3. (5 points) One of the inequalities we covered in class is as follows:

\[
\frac{\|\Delta x\|}{\|x\|} \leq \kappa(A) \frac{\|\Delta b\|}{\|b\|}.
\]

Explain what this inequality means. Label the various types of error as forward error or backward error.
4. (10 points; 5 points each)

(a) (5 points) The next number after 61 in a base-2 floating point number system is 62. What is the precision?

(b) (5 points) If $[L, U] = [-3, 6]$, what is the value of overflow for this system?
5. (10 points)

(a) (8 points) Compute the LU factorization of the following matrix:

\[
\begin{pmatrix}
1 & 2 & 4 \\
3 & 8 & 14 \\
2 & 6 & 13
\end{pmatrix}.
\]

(b) (2 points) To which algorithm is LU factorization equivalent?
6. (15 points) Let \( n \) be a positive even integer. Define the \( n \times n \) matrix \( A \) as follows:

\[
A_{ij} = \begin{cases} 
  n - |i - j| & \text{if } i \leq n/2 \text{ and } j \leq n/2 \\
  n - |i - j| & \text{if } i > n/2 \text{ and } j > n/2 \\
  1 & \text{if } i \leq n/2, j > n/2, \text{ and } j = i + n/2 \\
  1 & \text{if } i > n/2, j \leq n/2, \text{ and } i = j + n/2 \\
  0 & \text{otherwise.}
\end{cases}
\]

Write an efficient Matlab code to set-up \( A \) as a function of \( n \) by completing the following function prototype. **Hint:** You are encouraged to first work out the case when \( n = 6 \). This will help you determine the general pattern. Once you’ve done this, then write your Matlab code.

```matlab
function [A] = SetMeUp(n) 
% Input: n is a positive even integer.
%
% Output: A is the matrix described above.

A = zeros(n,n);
```

```matlab
function [A] = SetMeUp(n) 
% Input: n is a positive even integer.
% Output: A is the matrix described above.

A = zeros(n,n);
```
OPTIONAL: Extra-Credit Question

(10 points) Let $B$ be an $n \times n$ invertible matrix, and let $r, s, u, v, w,$ and $x$ be an $n \times 1$ vectors. Write a pseudocode which shows how to compute the following quantity efficiently and without using inverses:

$$Buv^Tw + u^TB^Tsr + B^{-1}x.$$