The rules for this exam are as follows:

- Write your name on the front page of the exam booklet. Initial each of the remaining pages in the upper-right hand corner. Sign the front of the exam booklet. Your exam will not be graded if you have not signed the front page of the booklet.
- This exam will last for 50 minutes.
- Show ALL work for partial/full credit. This includes any definitions, mathematics, figures, etc.
- The exam is closed book and closed notes.
- No calculators, laptops, ipads, or other types of non-medical electronic devices are allowed.
- No collaboration of any kind is allowed on the exam.

1. ______ (10 points)  
2. ______ (10 points)  
3. ______ (10 points)  
4. ______ (10 points)  
5. ______ (10 points)  
6. ______ (10 points)  
EC. ______ (6 points)  
T. ______ (60 points)
1. (10 points)

(a) (8 points) Let $m = 5$. Insert 23, 41, 68, 21, and 55 using linear probing and the hash function $h(x) = x \mod m$, into an initially empty hash table. Perform rehashing when the load factor is greater than 0.5. (Be sure to show all of your calculations, the table before rehashing, and the table after rehashing.)

(b) (2 points) What is the main advantage of rehashing?
2. (10 points; 2 points each) Mark **TRUE** or **FALSE** next to each question.

(a) It is better to use the last 3 digits of the phone number than the first 3 digits of the phone number in the design of a hash function.

(b) Rehashing should be performed when $\lambda > 0.5$ for open hashing with separate chaining.

(c) Keys can always be inserted whenever there are empty buckets when performing hashing with quadratic probing.

(d) In the context of double hashing, two hash functions are used to resolve collisions.

(e) One of the advantages of closed hashing is that it avoids the use of an additional data structure.

3. (10 points) A full node in a $k$-ary tree is one which has $k$ children.

Prove the following statement: The number of full nodes in a non-empty binary tree is always one less than the number of leaves.

**Hint:** One way to prove this is by induction.
4. (10 points) Prove that 

\[(2 + (-1)^n)n^2 = \Theta(n^2).\]

5. (10 points) Construct the (unique) binary tree corresponding to the given pair of tree traversals if possible. (If no such tree is possible, construct as much of the tree as possible, indicate where the breakdown lies, and state that it is not possible.)

Inorder: D, G, B, H, E, A, F, I, C

6. (10 points) Illustrate the data structure for the following tree using the left-child list-of-siblings implementation.
OPTIONAL: Extra-Credit Question

(6 points) Can a (unique) binary tree be reconstructed from a left-child list-of-siblings implementation diagram? Why or why not? Give a detailed example which clearly demonstrates your answer and explain your reasoning.