Questions:

1. (15 points)
   (a) (5 points) Given a set of 4 records with keys \( \{x_1, x_2, x_3, x_4\} \), where \( x_1 < x_2 < x_3 < x_4 \). Construct all possible binary search trees (BSTs) that can be used to store \( S \). **You must illustrate all of your trees clearly for credit.**
   (b) (10 points) Now suppose the probabilities of the 4 keys are as follows: \( p_1 = 0.2, p_2 = 0.6, p_3 = 0.15, p_4 = 0.05 \). Compute an optimal BST and determine its cost using the technique presented in class.

2. (30 points; 15 points each) Given a set \( S \) of 15 records with priorities \( \{5, 12, 15, 8, 9, 4, 1, 18, 6, 16, 11, 8, 12, 25, 9\} \).
   (a) Build a min 2-heap for \( S \) by inserting the records, in the order given, into an initially empty heap. When done, delete min twice.
   (b) Build a max 2-heap for \( S \) by using the bottom-up approach. When done, delete max twice.
   **You must illustrate all of your trees clearly for credit.**

3. (30 points; 15 points each) Given a set \( S \) of 15 records with priorities \( \{5, 12, 15, 8, 9, 4, 1, 18, 6, 16, 11, 8, 12, 25, 9\} \).
   (a) Build a minmax heap for \( S \) by inserting the records, in the order given, into an initially empty heap. When done, perform deleteMin and then deleteMax once.
   (b) Build a maxmin heap for \( S \) using the bottom-up approach. When done, perform deleteMax and then deleteMin once.
   **You must illustrate all of your trees clearly for credit.**

4. (10 points) **Scheduling of a Student’s Activities**
   (a) (5 points) Describe how a priority queue could be used to determine how to schedule the activities in a student’s life. Consider activities such as doing homework, studying for an exam, watching a sporting event, doing laundry, eating lunch, and attending a religious service. Which type of priority queue should be used? What are the relevant operation(s)?
   (b) (5 points) How does your answer change based on the addition of an unplanned event? Say, for example, that your friend calls you up and wants to know if you can leave in 15 minutes to go to a movie with her, and you’ve already set the schedule for your entire day.