The rules for this exam are as follows:

- Write your name on the front page of the exam booklet. Initial each of the remaining pages in the upper-right hand corner. Sign the front of the exam booklet. Your exam will not be graded if you have not signed the front page of the booklet.

- This exam will last for 50 minutes.

- Show **ALL** work for partial/full credit. This includes any definitions, mathematics, figures, etc.

- The exam is closed book and closed notes.

- No laptops, ipads, or other types of non-medical electronic devices are allowed.

- Calculators are allowed provided that they are only used to perform basic computations (and not programmed with algorithms or notes, for example).

- No collaboration of any kind is allowed on the exam.

1. ______ (10 points) 5. ______ (10 points)
2. ______ (10 points) 6. ______ (10 points)
3. ______ (10 points) EC. ______ (6 points)
4. ______ (10 points) T. ______ (60 points)
1. (10 points) Consider double hashing.

(a) (8 points) Insert 18, 41, 22, 44, 59, 32, 31, and 73 into an initially empty hash table using the initial hash function \( h(x) = x \mod m \), where \( m = 13 \) and the second hash function \( h^+(x) = R - (x \mod R) \), where \( R = 7 \). **Show your calculations and collision resolutions.**

(b) (2 points) In looking at the hash table, what is the main disadvantage you see for this hashing strategy?
2. (10 points) Prove the following statement: An extended binary tree with $n$ internal nodes has $n + 1$ external nodes.

**Hint:** One way to prove this is by induction on $n$.

3. (10 points) Suppose that the runtime for Algorithm 1 is $O(\log n)$, and the runtime for Algorithm 2 is $O(n)$. Which algorithm is faster and why? **Give a detailed answer.**
4. (10 points) Using the definition of big-O, prove that
\[
\frac{5n^4 - 2n^3 + \log n + 1}{n^2 - n} = O(n^2).
\]

5. (10 points) Construct the (unique) binary tree corresponding to the given pair of tree traversals if possible. If no such tree is possible, construct as much of the tree as possible, indicate where the breakdown lies, and state that it is not possible.

Postorder: T, Q, S, D, E, A, M, C, F, R, X, P
Inorder: T, S, Q, A, E, D, P, M, X, C, R, F.
6. (10 points) Illustrate the data structure for the following tree using the parent pointer implementation.
OPTIONAL: Extra-Credit Question

(6 points) Explain how a hash table can be used during a 911 emergency call.