The rules for this exam are as follows:

- Write your name on the front page of the exam booklet. Initial each of the remaining pages in the upper-right hand corner. Sign the front of the exam booklet. Your exam will not be graded if you have not signed the front page of the booklet.

- This exam will last for 50 minutes.

- Show **ALL** work for partial/full credit. This includes any definitions, mathematics, figures, etc.

- The exam is closed book and closed notes.

- No calculators, laptops, ipads, or other types of non-medical electronic devices are allowed.

- No collaboration of any kind is allowed on the exam.

1. ______ (15 points)
2. ______ (10 points)
3. ______ (10 points)
4. ______ (20 points)
5. ______ (10 points)
6. ______ (10 points)
7. ______ (10 points)
EC. ______ ( 8 points)
T. ______ (85 points)
1. (15 points; 3 points each) If possible, give an example of each of the following. If not possible, write “NOT POSSIBLE”. Do NOT include any additional justification.

   Note: Your answer for each subpart is worth 0 or 3 points.

   (a) The name of the computation model upon which complexity analysis is built.

   (b) The name of a closed hashing method which leads to less primary clustering when compared to hashing with linear probing.

   (c) A real-world application of hashing which we discussed in our in-class group activity.

   (d) A complete binary tree.

   (e) An optimal binary search tree containing two keys and their probabilities.
2. (10 points) By assuming that all basic operations require the same constant cost $C$, compute the cost of the resource function, $R_w(n)$, in closed-form for the following program segment using the simplified approach as discussed in class:

```plaintext
x = 951;
y = 237;
for i = 1 to n^2 do
    for j = 1 to i do
        y = x * y / 2 + 591;
    endfor;
x = 3 * y - 4 * x;
endfor;
```

3. (10 points) Using the definition of big-O, prove that

$$
\frac{5n^4 - 2n^3 + \log n + 1}{n^2 - n} = O(n^2).
$$
4. (20 points) Closed hashing with quadratic probing.

(a) (5 points) Define the load factor $\lambda$ of a hash table with $m$ buckets and $n$ stored elements.

(b) (10 points) Using the hash function $x \mod m$ and quadratic probing, construct a hash table $H$ with $m = 7$ buckets by inserting a set of 6 records with keys \{76, 93, 40, 35, 8, 15\}, in the given order, into $H$.

(c) (5 points) What happens if you next try to insert the key 47 into $H$? Illustrate the problem that occurs.
5. (10 points) Construct the (unique) binary tree corresponding to the given pair of tree traversals if possible. If not such tree is possible, state that is the case.

Preorder: 7, 1, 0, 3, 5, 4, 6, 9, 8
Inorder: 0, 1, 3, 4, 5, 6, 7, 8, 9.

6. (10 points) Illustrate the data structure for the following tree using the left-child list-of-siblings pointer implementation.
7. (10 points; 2.5 points each) Given a set of 1,297 records to be stored using a binary search tree $T$. Specify an integer solution to each of the following questions if possible.

(a) What is the minimum height of $T$?

(b) What is the maximum height of $T$?

(c) What is the minimum number of leaves of $T$?

(d) What is the maximum number of leaves of $T$?
OPTIONAL: Extra-Credit Question

(a) (8 points) Consider the following probability table, where $p_i$ is the probability of key $k_i$:

<table>
<thead>
<tr>
<th>$i$</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_i$</td>
<td>$k_1$</td>
<td>$k_2$</td>
<td>$k_3$</td>
</tr>
<tr>
<td>$p_i$</td>
<td>0.1</td>
<td>0.3</td>
<td>0.6</td>
</tr>
</tbody>
</table>

Given: $k_1 < k_2 < k_3$.

Construct the optimal binary search tree based on this information.