Questions:

1. (12 points) Assuming that all basic operations require the same constant cost $C$, and by concentrating on the dominating step(s), compute the cost of the resource function, $R(n)$, for the following program segment:

   
   ```plaintext
   x = 3;
   y = -1;
   for i = 1 to n do
     for j = 1 to n do
       for k = 1 to j^2 do
         x = x * y + 2;
       endfor;
     endfor;
     for m = 1 to i^3 do
       y = x + 2 * m * i + j^2;
     endfor;
   endfor;
   ```

2. (12 points) When implementing an ADT for a set of data $S$, $|S| = 3^{15}$, it is determined that a search operation, search(x,S), requires 0.004 seconds to execute. If the complexity of the search operation is given by the following closed-form expressions $T(n)$, compute the time required to execute this operation when $|S| = 3^{30}$.

   (a) $T(n) = \log n$.
   (b) $T(n) = 142n$.
   (c) $T(n) = n^2 + 3n - 2$.
   (d) $T(n) = 3^n$.

3. (10 points) Consider two algorithms $A_1$ and $A_2$ with closed-form complexity $T_1(n) = 5n^2$ and $T_2(n) = 314n - 12$. Find the smallest integer $n_0$ such that for all $n > n_0$, algorithm $A_2$ will always be better than algorithm $A_1$.

4. (10 points) Prove or disprove the following statement using the definition of big-$\Theta$.

   $$\frac{n^3 - 6n^2 + 7n - 2\log n}{n^2 - 2n + 6\log n - 1} = \Theta(n).$$
5. (10 points) Prove or disprove the following statement using the definition of big-$O$:

$$3^n = O(2^{2n}) = O(3^{3n}).$$

6. (12 points) Prove that

$$\frac{n^4 - 168n^3 + 18n^2 - 15n}{18n^2 - 2n + 1024} \neq \Omega(n^3).$$

7. (12 points) Given an array $A = [a_1, \ldots, a_n]$ and a key $x$. Assuming that $Pr(x = a_i) = \frac{3}{5n^2 - 3}, \forall i$. Based on the number of comparisons between $x$ and $A[i]$'s, compute $T_a(n)$ in closed-form if the sequential search algorithm is used for search for $x$ in $A$ (starting at $A[1]$).

8. (15 points) Do Exercise 5.1(parts a through c only) on p. 237 of your textbook.