Lab #4
Transformers

For this lab, one person from your group will need to check out the following from the shop:

- A probe kit (Metal toolbox)
- LCR Meter
- Audio transformer (11.5:1): Mouser Electronics, No. 42TM013

Electronic components required for this experiment

- Audio transformer (11.5:1), Mouser Electronics, No. 42TM013
- 8.2 Ω resistor
- 1 kΩ resistor
- 68 nF capacitor

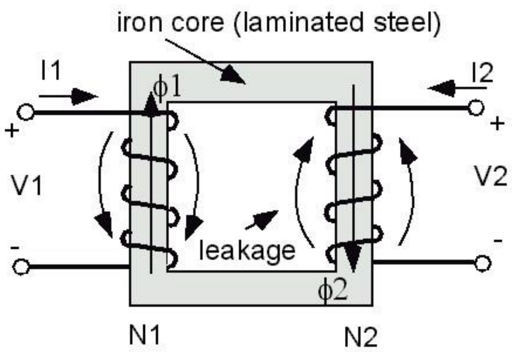
Lab Objectives

- Comparison of the ideal transformer versus the physical transformer
- Measure some of the circuit parameters of a physical transformer to determine how they affect transformer performance.

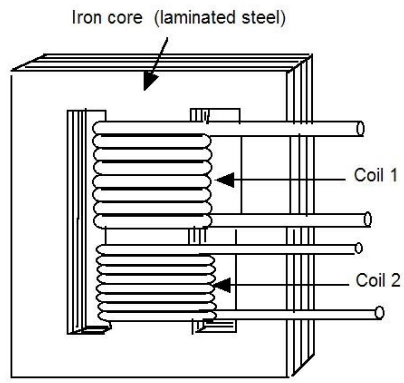
Background

Transformers

A transformer is a specific form of a coupled circuit in which the mutual coupling between two coils is deliberately made strong. In most cases, a common magnetic flux path is provided by an iron core. A transformer can be represented as shown in Figure 1a. A physical implementation is given in Figure 1b.



(a)



(b)

Figure 1 (a) An iron core transformer showing magnetic paths.

Figure 1 (b) Construction of an iron core transformer. For clarity the coils are shown separated, Physically, one coil is usually wound around the second coil to maximize the magnetic coupling

Here, N1, N2 are the number of turns at the primary and secondary windings; φ_1 is the flux produced by I_1 and φ_2 is the flux produced by I_2. In Figure 1a, according to the right-hand rule method for magnetic fields, the fluxes produced by the currents are additive when I_1 and I_2 have the same sign. If the either the primary or secondary windings are reversed (e.g. from left-hand to right-hand), the fluxes would be subtractive.

When coil 1 is supplied with a time-varying current, the magnetic field is coupled into coil 2 which induces a voltage V_2 across the coil. The resultant current in coil 2 creates its own magnetic field which, in turn, is coupled to coil 1. This mutual coupling is in the form of a mutual inductance, *M*. The mutual inductance *M* is related to the self-inductances L_1 and L_2 by:

$$M = k\sqrt{L_1 L_2} \text{ (where } k \text{ is the coupling factor and } k \le 1\text{)}$$

The Ideal Transformer

Figure 2 shows a typical transformer circuit. Here, the left winding is driven by a source and is considered to be the primary, and the right side has a load attached and is considered to be the secondary. In addition, the dots associated with the windings are define the dot convention, which indicates that magnetic fluxes produced positive currents into the dotted terminals are additive. Reversing the dot on either inductor would result in changing the sign of the resulting value of M.

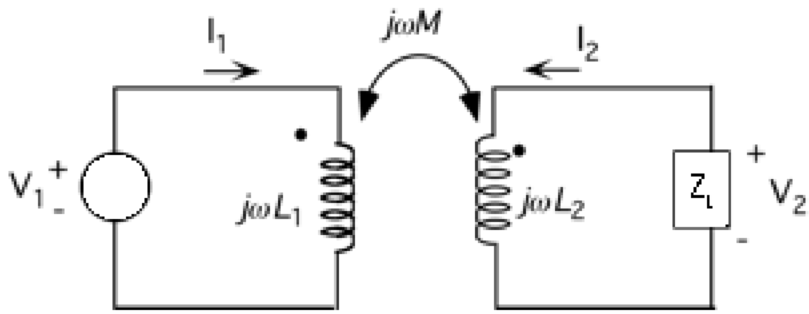


Figure 2: Couped Inductors

From KVL around both loops, we obtain:

$$V_1 = jwL_1I_1 + jwMI_2 \quad (1)$$

$$V_2 = jwMI_1 + jwL_2I_2 \quad (2)$$

where $M = k\sqrt{L_1L_2}$

From Ohm's law, we also have $V_2 = -Z_L I_2$. Substituting and rearranging, we find the following relationships between the voltages and currents:

$$\frac{V_1}{V_2} = \frac{1}{k}\sqrt{\frac{L_1}{L_2}} + \frac{jW}{Z_L}\frac{\left(1-k^2\right)}{k}\sqrt{L_1 L_2} \qquad (3)$$

$$\frac{I_1}{I_2} = \frac{-jWL_2 - Z_L}{jWk\sqrt{L_1 L_2}} \qquad (4)$$

As $k \to 1$, the voltage expression becomes:

$$\frac{V_1}{V_2} = \sqrt{\frac{L_1}{L_2}} = \frac{N_1}{N_2} \qquad \text{(when } k \to 1\text{)} \qquad (5)$$

where N_1 and N_2 are the number of turns in each inductor. Thus, when the inductors are maximally coupled, the ratio of the winding voltages equals the turns ratio, independent of the load impedance Z_L.

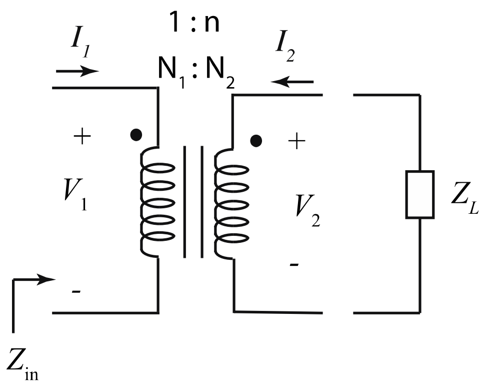
If $|jWL_2| >> Z_L$ the current expression becomes:

$$\frac{I_1}{I_2} = -\sqrt{\frac{L_2}{L_1}} = -\frac{N_2}{N_1} \qquad \text{(when } |jWL_2| >> Z_L \text{ and } k \to 1\text{)} \qquad (6)$$

Further, when both $|jWL_2| >> Z_L$ and $k \to 1$, the reflected impedance looking into the primary is:

$$Z_1 = \left(\frac{N_1}{N_2}\right)^2 Z_L \qquad \text{(when } |jWL_2| >> Z_L \text{ and } k \to 1\text{)} \qquad (7)$$

Equations 5-7 define the operation of an *ideal transformer*. Here, we see that when the conditions $\left| jwL_2 \right| >> Z_L$ and $k \to 1$ are met, the self and mutual reactive impedances of the transformer effectively "disappear" and the voltage and current ratios and the impedance reflected into the primary are controlled only by the coil turns ratios. These relations are summarized in Figure 3:



Ideal Transformer Relations

$$\frac{V_1}{V_2} = \frac{N_1}{N_2} = \frac{1}{n} \qquad \frac{I_1}{I_2} = -\frac{N_2}{N_1} = -n$$

$$Z_{in} = \left(\frac{N_1}{N_2} \right)^2 Z_L$$

Figure 3: The Ideal Transformer

Non-ideal Transformers

In order for coupled inductors to be considered an ideal transformer, three conditions must be met:

1) The coupling factor is unity
2) The self-impedances of the transformer coils are much larger than the load impedance
3) The losses in the coils and core are negligible

Practical transformers can only attempt to approximate these conditions. To see the effect of the primary self-inductance L1 not being much larger than the reflected load impedance from the secondary, we note that for k=1, equations 5 and 6 can be rearranged to show that the admittance looking into the primary terminals is:

$$Y_1 = \frac{I_1}{V_1} = \frac{1}{jwL_1} + \left(\frac{N_2}{N_1} \right)^2 \frac{1}{Z_L} \qquad (k \to 1) \qquad (8)$$

The first term on the right is the admittance of the primary inductance L_1, often called the magnetizing inductance (L_m). The second term is the reflected admittance from the secondary of an ideal transformer with the same turns ratio. Hence, the magnetizing inductance will appear as a shunt in parallel with an ideal transformer.

Next, there is always a "leakage flux" that that is not shared by the primary and secondary windings. This means that part of both the primary and secondary self-inductances do not participate in the transformer-action and will simply appear as parasitic series inductors L_a and L_b in the primary and secondary circuits, restively.

Also, there are resistive losses in both the winding wires, as well as magnetic losses in the core. The wire resistances appear as series resistors R_a and R_b the primary and secondary circuits, respectively. The core losses, although magnetic, will appear as a resistive shunt Rm across the primary terminals,

Finally, there is always capacitive coupling in each winding, which appear as shunt capacitances C_a and C_b across the primary and secondary sides of the transformer.

The combined effects of losses, non-unity coupling, and stray capacitances can be added to yield the following equivalent circuit of a practical, nonideal transformer and are shown in Figure 4.

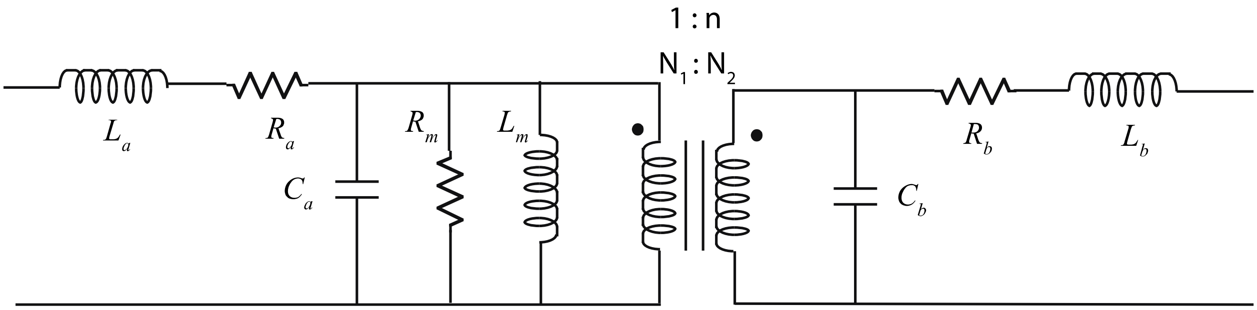


Figure 4: Equivalent circuit of non-ideal transformer

R_a = primary winding resistance
L_a = primary *leakage* inductance
C_a = primary winding capacitance
R_b = secondary winding resistance
L_b = secondary *leakage* inductance
C_b = secondary winding capacitance
R_m represents core losses (hysteresis and eddy current losses)
L_m = magnetizing/primary inductance

The nonideal (parasitic) components can be reflected onto the primary side of the circuit to yield:

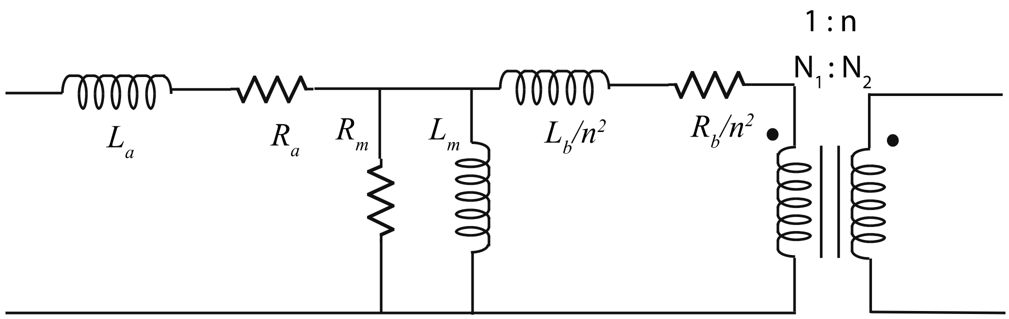


Figure 5: Equivalent Circuit of non-ideal transformer (ignoring capacitance) with parasitic elements all in the primary circuit

Here, R_a and R_b are the resistances of the primary and secondary windings, respectively. The losses in the core are represented by the resistor R_m, and L_m is the self-inductance of the primary winding (the *magnetizing inductance*). L_a is the leakage inductance, the result of flux generated by the primary coil that does not link with the secondary coil. Typically, $L_a \ll L_m$.

The capacitors C_a and C_b represent the winding capacitances that are very small so that their impedance can usually be neglected in the mid-frequency range of operation. Since the circuit contains both capacitive and inductive components (RLC circuit) a condition for resonance exists that significantly alters the circuit behavior at high frequencies.

Finally, since the shunt impedances are generally much larger than the series impedances, the series imped aces can be combined, resulting in:

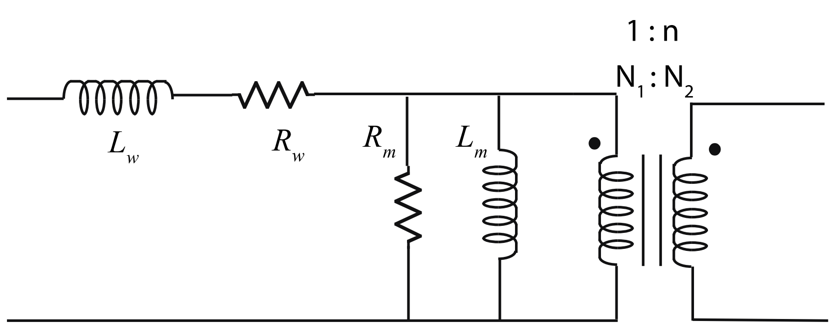


Figure 6: Simplified equivalent circuit of transformer at low frequencies

Experimental Procedure

The transformer to be used in this lab session is pictured in Figure 7a. It is typically used in the audio frequency band (200 Hz to 5 kHz) as an impedance match between a high impedance audio amplifier and a speaker (typically 8 Ohms). Either coil can be energized. If the 11.5x coil is energized, we have a 11.5:1 step-down transformer; if the 1x coil is used as the primary (energized coil) we have a step-up transformer.

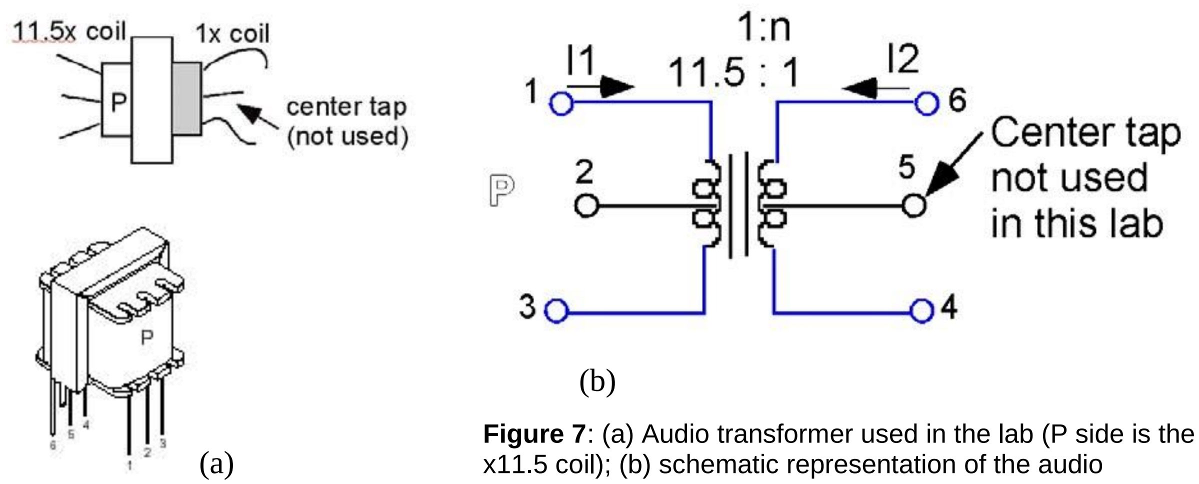


Figure 7: (a) Audio transformer used in the lab (P side is the x11.5 coil); (b) schematic representation of the audio transformer with center taps. For the lab, we won't use the center taps.

Experiment 1: Measurement of the transformer parasitic parameters with RLC Meter

To measure the parasitic parameters of a real (non-ideal) transformer you will use an LCR meter (provided by the GTA). This meter is used to measure inductance, capacitance, and resistance.

In the inductance mode, the meter measures the series (default) or parallel inductance and *Q* of inductors at 1 kHz (default) or 120 Hz. Using these values, the series or parallel equivalent circuit of a real inductor can be determined:

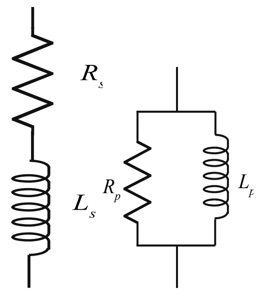


Figure 8: Lossy Inductor equivalent circuits in series and parallel

The relationships between series and parallel inductances and loss resistances with the component Q are given by:

$$Q = \frac{2\pi f L_s}{R_s} = \frac{R_p}{2\pi f L_p} \quad (9)$$

$$\frac{R_p}{R_s} = 1 + Q^2 \quad (10)$$

$$\frac{L_p}{L_s} = \left[1 + \left(\frac{1}{Q}\right)^2\right] \quad (11)$$

Although the transformer also has leakage capacitance, these values are small enough to have negligible affects at audio frequencies.

Place the transformer on a breadboard. You will be using the transformer as a step-down transformer, so consider the x11.5 coil as the primary. The parameters L_w, R_w, L_m and R_m (see Figure 7b) can be determined from short-circuit test and open-circuit tests:

<u>Step 1: Short the secondary coil</u>

This will reflect as a short across the x11.5 primary coil, so that the LCR meter will measure the leakage reactance Lw and winding resistance Rw.

Using the function key, set the LCR to the inductance mode. Next, use the frequency key to set the measurement frequency to 1 kHz. Use test clips to attach the transformer terminals to the LCR input terminals.

In its default mode, the LCR default mode measures the series inductance L_W and quality factor Q, from which the series resistance Rw can be found. Make sure the meter is displaying Q, not D (where D = 1/Q is the *dissipation factor*.) A single press of the D/Q button switches between the two display modes.

<u>Step 2: Open circuit the secondary coil</u>

Since the values of Lm and Rm are much larger than those of Lw and Rw, respectively, the open circuit impedance looking into the primary is basically the parallel combination of Lm and Rm.

Using the function key, set the LCR to the inductance mode. Next, use the frequency key to set the measurement frequency to 1 kHz. Use test clips to attach the transformer terminals to the LCR input terminals.

Since Lm and Rm are a parallel combination, you can either use the default mode of the LCR to obtain the series inductor and resistor values and convert these to parallel component values. Or, you can set the LCR meter to the parallel mode. To do this: press the DH key and then press and hold D/Q button for one second. When the "PAL" character appears on the secondary display, the meter is in in the parallel mode. Press D/Q again to exit and return to the default mode.

Step 3: Primary measurements

This particular transformer has center-taps for both the primary and secondary. Measure the inductance of both halves of the primary separately with the secondary open-circuited. Compare these values with the total inductance you found in step 1. Why is the total inductance not equal to the sum of the two halves?

Experiment 2: Measurements of current and voltages in the transformer circuit

In this experiment, you will build the circuit shown in Figure 9. Here, the 8.2 Ω load resistance R_L is similar to the load of a speaker, and the 1 KΩ , which simulates the output resistance of a typical audio amplifier. Adjust the output of the function generator for a sinusoid of 5 Vrms and 1kHz. Measure the actual values of the resistors before assembling the circuit.

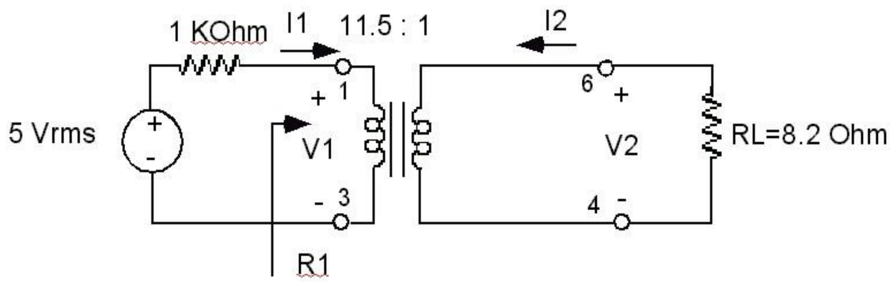


Figure 8: Circuit with a 11.5:1 audio transformer and load of 8.2 Ω

Step1

Measure the actual values of the two resistors with the LCR meter (set to the R mode) before placing them on the breadboard.

Step 2

Since we are interested in both magnitude and phase, use the external trigger terminal of the oscilloscope to the top terminal of the transformer and set the oscilloscope to external trigger mode.

Step 3

Measure the voltage V1 and V2 (in V rms) over the primary and secondary coils, respectively. Observe the phase of V2 in reference to V1. Also, compare the measured value of V1/ V2 (magnitude and phase) with that according to the turns ratio alone.

Step 4

Using two measurement probes, use the difference mode of the oscilloscope the voltage across the 1 kΩ resistor and calculate the corresponding current I_1 in the primary coil. Find also the current I_2 in the secondary loop.

Calculate the current ratio and compare it to the voltage ratio. Compare the measured I_1/I_2 ratio (magnitude and phase) with that calculated using just the turns ration.

Step 5

Based on the measurement of V_1 and I_1 what is the resistance seen at the input of the primary coil? How does this compare with what you would expect if the transformer was ideal?

Step 6

Use the measured values of current and voltage to calculate the power delivered by the function generator and the power absorbed by the two resistors. Compare the generated and dissipated power. Explain the difference.

Step 7

Use PSpice to simulate this experiment using the measured values of the parasitic elements of the transformer. Compare the values of V_1/V_2 and I_1/I_2 from the PSpice model with the measured values.

Step 8

Comment on whether the parasitic components have an appreciable effect on the circuit performance.

***Note:** for the calculations you can use RMS or amplitude values. Be consistent in your calculations.

Transformer frequency response

A good audio transformer should have a constant voltage ratio over it specified frequency range. The manufacturer gives a frequency response of 300 Hz to 3.4 kHz with a variation of ±3dB (note: dB corresponds to $20*\log_{10}A$).

As discussed earlier, at very high frequencies the effect of the capacitors will be seen as a resonance effect that will cause the ratio V_2/V_1 to increase considerably above the nominal value n (=1/11.5).

The goal of this experiment is to measure the frequency response over a large frequency range and verify that the response is within the specification of the manufacturer. You will also measure the response at very high frequencies and measure the resonant frequency. This will allow you to find the parasitic capacitance.

The equivalent model of the transformer can now be approximated as shown in Figure 10. Here, Cr includes the primary and reflected secondary capacitance: $Cr = Ca + n^2Cb$.

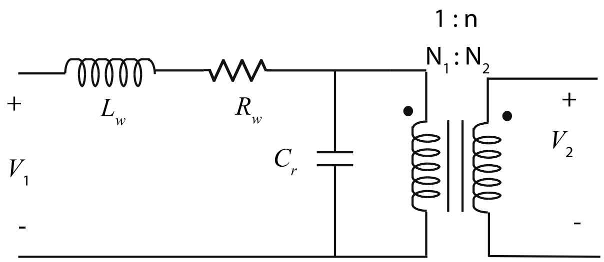


Figure 10: Transformer equivalent circuit at high frequencies

When the secondary is open circuited, the secondary will present no load across the capacitor. As the frequency increases, the capacitor admittance will dominate that of the magnetizing inductor and resistance (not shown), so the primary circuit will approximate a series RLC network.

At resonance, the capacitor and inductor impedances will cancel, causing the current to peak, and, since the circuit *Q* is high, the voltage across C_r and, therefore and V_2, will peak. The resonant frequency is related to L_w and C_r by:

$$f_r = \frac{1}{2p\sqrt{LwCr}} \qquad (12)$$

<u>Experiment 3: Measuring Frequency Response</u>

<u>Step 1</u>

Connect a function generator to the x11.5 coil and leave the secondary open (RL = ¥). Set the generator frequency to 1 kHz sine waveform and adjust the output of the function generator to 10 Vp-p. Measure the secondary voltage.

<u>Step 2</u>

Over the frequency range f = 100 Hz to 5 MHz, measure and plot amplitude and phase of the V_2/V_1 ratio in half-decade frequency steps.

<u>Step 3</u>

You will notice that at high frequencies (MHz range) the output voltage starts to increase quickly. This is a result of the resonance due to the inductance L_w and the winding capacitance C_r. Increase the number of measurements around this resonant frequency f_o so that you can plot the frequency response accurately around this peak. Plot $20\text{Log}|V_2/V_1|$ (in dB) vs frequency.

<u>Step 4</u>

Now you will add a capacitor of 68nF over the secondary terminals and measure the frequency response of |V2/V1|, similarly as you did in the previous section. Since you add a larger capacitor than the parasitic winding capacitances, you can expect that the resonant frequency will be smaller.

a. Select the x11.5 winding as the primary side of the transformer and set V1 = 10 V p-p at a frequency of 1 kHz.

b. Load the secondary with the 68 nF capacitor. Vary the frequency from 100 Hz to about 5MHz and record V1 and V2. Increase the number of measurements around the resonant frequency.

c. Plot the frequency response of 20Log|V2/V1| (in dB) vs. frequency (log scale) This plot can be drawn on the same graph as the previous one. Compare the location of the resonant frequency.