EECS 461 Short Quiz #2

Probability and Statistics
February 26, 2008

Name: KEY

Closed Book and Closed Notes: Show all work. Provide numerical answers as four-place demimals.

1. (30 %) A player of a video game is confronted with a series of opponents and has an 80% probability of defeating each one. Success with any opponent is independent of previous encounters. The player has only one "life," and continues to contest opponents until he is defeated.

   (a) What is the pdf of the number of opponents contested in a game?

   (b) What is the probability that a player defeats at least two opponents in a game?

   (c) What is the expected number of opponents contested in a game?

   (d) What is the probability that a player contests four or more opponents in a game?

   (e) How does this problem change if the player has more than one "life"?

(a) We have a Bernoulli trial ("success" = defeat, "failure" = victory) $p = 0.20$
We are "waiting" for our first "success" (our first defeat) $q = 0.80$
X is a geometric RV $\boxed{P(X=x) = (0.80)^{x-1}(0.20),\ x \geq 1}$

(b) defeat at least 2 opponents $\rightarrow 3 \leq X \leq \infty$

$$P(3 \leq X \leq \infty) = 1 - P(1 \leq X \leq 2) = \sum_{x=1}^{2} (0.8)^{x-1}(0.2) \boxed{= 0.6400}$$

(c) X = # of opponents contested in a game, $E(X) = \mu_X = \frac{1}{p} \boxed{= 5}$

(d) contests 4 or more opponents $\rightarrow 4 \leq X \leq \infty$

$$P(4 \leq X \leq \infty) = 1 - P(1 \leq X \leq 3) = \sum_{x=1}^{3} (0.8)^{x-1}(0.2) \boxed{= 0.5120}$$

(e) If we have r "lives," then this problem changes to "waiting" for our r-th "success" (our r-th defeat) $\Rightarrow$ $\boxed{\text{negative binomial RV}}$

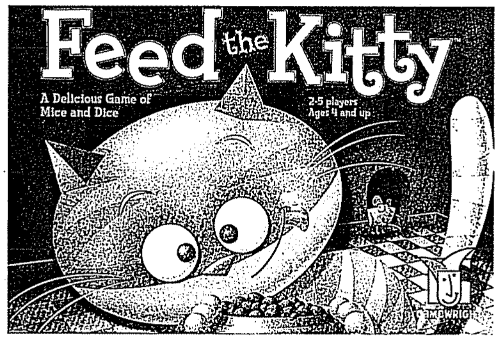
2. (20 %) A loom experiences one yarn breakage approximately every 10 hours. A particular style of cloth is being produced that will take 25 hours on this loom. If three or more breaks are required to render the product unsatisfactory, find the probability that this style of cloth is finished with acceptable quality.

We have a Bernoulli trial ("success" = break, "failure" = no break)
and we can't really enumerate how many trials there are (n)
and we cant really count the "no breaks" (n-x), all we know is
the average rate of success $\Rightarrow$ this is a poisson process

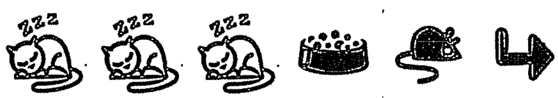
1 break/10 hours = 2.5 breaks/25 hours $\Rightarrow \lambda = 2.5$

$$P(\text{cloth finished with acceptable quality}) = P(0 \leq X \leq 2) = \sum_{x=0}^{2} e^{-2.5}\frac{(2.5)^x}{x!} \boxed{= 0.5438}$$

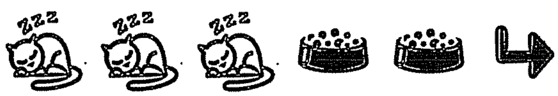
3. (50 %) Ready or not, here we go! You are about to learn how to play *Feed the Kitty*. We'll keep it simple:



   - There are **2** players (you vs. your opponent).
   - You each start with a pile of **8** little wooden mice.
   - There is a bowl with a few extra wooden mice.
   - You each take turns rolling a **pair** of six-sided dice.
   - **Die #1** has the following six sides:



   - **Die #2** is slightly different and has the following six sides:



   - The markings on the dice have the following meanings:

 Do nothing.

 Take a wooden mouse out of your pile and put it in the bowl.

 Take a wooden mouse out of the bowl and put it in your pile.

 Take a wooden mouse out of your pile and give it to your opponent.

   - After you roll the dice, you take whatever action is indicated by Die #1 and Die #2.
   - The game ends when you or your opponent run out of mice.

Questions:

(a) Let the random variable X_1 be the number of mice you **lose** after you roll Die #1 (when X_1 is positive, you lose mice; when X_1 is negative, you gain mice). Specify the pdf of X_1.

(b) What is the mean of X_1? Provide a numerical value.

(c) What is the variance of X_1? Provide a numerical value.

(d) Let the random variable X_2 be the number of mice you **lose** after you roll Die #2 (when X_2 is positive, you lose mice; when X_2 is negative, you gain mice). Specify the pdf of X_2.

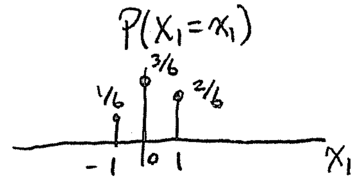
(e) What is the mean of X_2? Provide a numerical value.

(f) What is the variance of X_2? Provide a numerical value.

(g) How many rounds do we expect this game to last?

[Space to answer Problem 3]

(a) $P(X_1 = x_1) = \begin{cases} \frac{1}{6}, & x_1 = -1 \\ \frac{3}{6}, & x_1 = 0 \\ \frac{2}{6}, & x_1 = +1 \end{cases}$



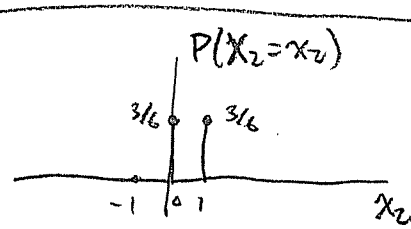
(b) $\mu_1 = E(X_1) = \sum_{x_1} x_1 P(X_1 = x_1) = (-1)\frac{1}{6} + (0)\frac{3}{6} + (1)\frac{2}{6} \boxed{= \frac{1}{6} = 0.1667}$

(c) $E(X_1^2) = \sum_{x_1} x_1^2 P(X_1 = x_1) = (-1)^2\frac{1}{6} + (0)^2\frac{3}{6} + (1)^2\frac{2}{6} = \frac{3}{6}$

$\sigma_1^2 = E(X_1^2) - \mu_1^2 = \frac{3}{6} - \left(\frac{1}{6}\right)^2 \boxed{= \frac{17}{36} = 0.4722}$

(d)

$P(X_2 = x_2) = \begin{cases} 0, & x_2 = -1 \\ \frac{3}{6}, & x_2 = 0 \\ \frac{3}{6}, & x_2 = +1 \end{cases}$



(e) $\mu_2 = E(X_2) = \sum_{x_2} x_2 P(X_2 = x_2) = (-1)0 + (0)\frac{3}{6} + (1)\frac{3}{6} \boxed{= \frac{1}{2} = 0.5000}$

(f) $E(X^2) = \sum_{x_2} x_2^2 P(X_2 = x_2) = (-1)^2 0 + (0)^2 \frac{3}{6} + (1)^2 \frac{3}{6} = \frac{1}{2}$

$\sigma_2^2 = E(X_2^2) - \mu_2^2 = \frac{1}{2} - \left(\frac{1}{2}\right)^2 \boxed{= \frac{1}{4} = 0.2500}$

(g) We can't overlook the random variables from our opponent's turn, because our pile of mice is affected by them

X_3 = # of mice we lose due to opponent rolling Die #1 $\mu_3 = E(X_3) = -\frac{1}{6}$

X_4 = " " " " " " " " " Die #2 $\mu_4 = E(X_4) = -\frac{1}{6}$

X_r = # of mice we lose per round $= X_1 + X_2 + X_3 + X_4$

$E(X_r) = E(X_1 + X_2 + X_3 + X_4) = \mu_1 + \mu_2 + \mu_3 + \mu_4 = \frac{1}{6} + \frac{3}{6} - \frac{1}{6} - \frac{1}{6} = \frac{1}{3}$

We lose 1/3 a mouse per round on average, We expect it will take $\boxed{24}$ rounds for someone to lose all 8 mice

Alternate viewpoint on Problem 3 part (g)

Since the ↳ side means you lose a mouse when you roll, and it means you gain a mouse when your opponent rolls, you could choose to view it as another "Do nothing" case on average, since ½ of the times it occurs correspond to a $+1$ and the other ½ correspond to a -1.

With this interpretation, a revised answer to part (b) is

$$E(X_1) = 0$$

and a revised answer for part (e) is

$$E(X_2) = \frac{1}{3}$$

and the expectation for the entire round is

$$E(X_r) = E(X_1 + X_2) = 0 + \frac{1}{3} \boxed{= 1/3}$$, which gives us $\boxed{24}$ rounds again

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One final comment, it is easy to see that nothing really changes when there are more than 2 players. You still give up mice to an opponent, and you still receive mice from an opponent. The only change is that you get less than 8 mice to begin with [see the rules for the details, I'm sure many of you will ;)]