# EECS 461 Short Quiz #1
Probability and Statistics
February 5, 2008

Name: ___KEY___

Closed Book and Closed Notes: Show all work. Provide numerical answers as four-place demimals.

1. (50 %) In a chemical plant, 24 holding tanks are used for final product storage. Four tanks are selected at random and without replacement. Suppose that six of the tanks contain material in which the viscosity exceeds the customer requirements.

    (a) What is the probability that none of the tanks in the sample contain high viscosity material?

    (b) What is the probability that exactly one tank in the sample contains high viscosity material?

    (c) What is the probability that at least one tank in the sample contains high viscosity material?

    (d) In addition to the six tanks with high viscosity levels, four different tanks contain material with high impurities. What is the probability that exactly one tank in the sample contains high viscosity material and exactly one tank in the sample contains material with high impurities?

$$\binom{24}{4} = 10626 \text{ possible samples}$$
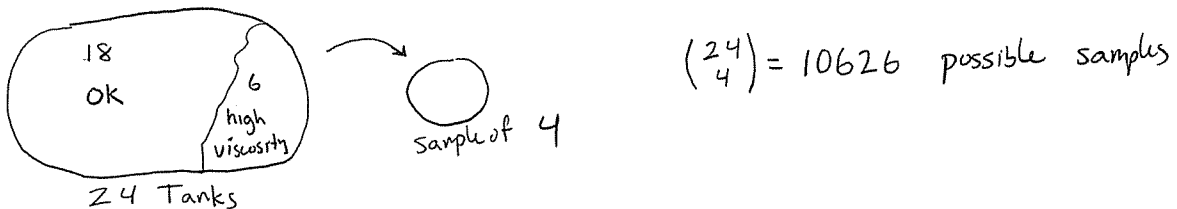
(a) There are $\binom{18}{4}\binom{6}{0} = 3060$ samples with 4 OK and 0 high viscosity

$$P(\text{exactly } 0) = \frac{3060}{10626} = \boxed{0.2880}$$

(b) There are $\binom{18}{3}\binom{6}{1} = 4896$ samples with 3 OK and 1 high viscosity

$$P(\text{exactly } 1) = \frac{4896}{10626} \boxed{= 0.4608}$$

(c) There are $\binom{18}{3}\binom{6}{1} + \binom{18}{2}\binom{6}{2} + \binom{18}{1}\binom{6}{3} + \binom{18}{0}\binom{6}{4} = 7566$ samples with at least 1 high viscosity.

$$P(\text{at least } 1) = \frac{7566}{10626} \boxed{= 0.7120} = 1 - P(\text{exactly } 0) = 1 - 0.2880 \boxed{= 0.7120}$$

(d)

There are $\binom{14}{2}\binom{6}{1}\binom{4}{1} = 2184$ samples with exactly one high viscosity and 1 high impurities

$$P(1 \text{ high viscosity and } 1 \text{ high impurities}) = \frac{2184}{10626} \boxed{= 0.2055}$$

2. (20 %) Of five cards, two are painted red on both sides; two are painted black on both sides; and one is painted red on one side and black on the other. A card is randomly chosen and placed on a table. If the side facing up is red, what is the probability that the other side is also red?

There are 5 cards, each with 2 sides, and we reach in and pick one of these $\binom{5}{1}\binom{2}{1} = 10$ sides. It is given that the side we picked is red. We can probably intuit our way to the correct answer of 0.8, but let's use Baye's rule instead
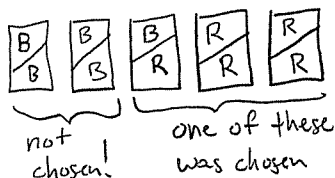
A_1 = Both sides are red
A_2 = Only one side is red
B = the side chosen is red

$$P(A_1|B) = \frac{P(B|A_1)P(A_1)}{P(B)} = \frac{(1)(2/5)}{1/2} = \frac{4}{5} \boxed{= 0.8}$$

3. (30 %) Customers are used to evaluate preliminary product designs. In the past, 95% of highly successful products received good reviews, 60% of moderately successful products received good reviews, and 10% of poor products received good reviews. In addition, 40% of products have been highly successful, 35% have been moderately successful, and 25% have been poor products.

(a) What is the probability that a product attains a good review?

(b) If a new design attains a good review, what is the probability that it will be a highly successful product?

(a) use the Law of total Probability
A_1 = product is highly successful
A_2 = product is moderately successful
A_3 = product is poor
B = product attains a good review

$$P(B) = \sum_{j=1}^{3} P(B|A_j)P(A_j) = (0.95)(0.40) + (0.60)(0.35) + (0.10)(0.25) \boxed{= 0.6150}$$

Use Bayes' Rule

$$P(A_1|B) = \frac{P(B|A_1)P(A_1)}{P(B)} = \frac{(0.95)(0.40)}{0.6150} \boxed{= 0.6179}$$