EECS 461 Short Quiz #3  
Probability and Statistics  
April 25, 2007  
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Closed Book and Closed Notes: Show all work. Provide numerical answers as four-place decimals.

1. (40%) A police crime lab suspects that the purity of *methamphetamine* in the local area has changed recently. In the past, the drug was on average 21.0 percent pure. In a random sample of twelve recent drug busts, the sample mean was 23.4 percent purity, with a sample standard deviation of 4.1 percent purity.
   a. Test the null hypothesis $H_0: \mu = 21$ against the alternate hypothesis $H_a: \mu \neq 21$ using $\alpha = 0.05$. Do you accept or reject the null hypothesis?
   b. This time, test the null hypothesis $H_0: \mu = 21$ against the alternate hypothesis $H_a: \mu > 21$ using $\alpha = 0.05$. Do you accept or reject the null hypothesis in this case?
   c. Calculate the $p$-value for the test in (a). State whether you accept or reject the null hypothesis in (a) by using the $p$-value.
   d. Calculate the $p$-value for the test in (b). State whether you accept or reject the null hypothesis in (b) by using the $p$-value.

\[ n = 12 \]
\[ \bar{x} = 23.4 \quad s^2 = (4.1)^2 \]

\[ t_{11,0.025} = -2.201 \]
\[ t_{11,0.025} = 2.201 \]

According to the table (and my calculator)

\[ t_{11} = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{23.4 - 21.0}{4.1/\sqrt{12}} = 2.0278 \]

This value is in between our two limits.

\[ t_{11,0.975} < t_{11} < t_{11,0.025} \] so accept $H_0: \mu = 21$

b)

\[ t_{11,0.05} = 1.796 \]

In this case the value we observe, $t_{11} = 2.0278$ exceeds the limit $t_{11} < t_{11,0.05}$ so

\[ \text{Reject } H_0: \mu = 21 \]

\[ \text{Accept } H_0: \mu > 21 \]

c)

\[ 0.0338 \]

The area under the curve to the right of $t_{11} = 2.0278$ is

\[ 0.0338 \] and this is a two sided hypothesis, so

\[ p = 2 \times 0.0338 = 0.0675 \] thus this test is not significant enough to overturn the null hypothesis in an $\alpha = 0.05$ test

d)

\[ 0.338 \]

In a one sided hypothesis we have $p = 0.0338$, which is significant enough to overturn the null hypothesis in an $\alpha = 0.05$ test
2. (40%) Consider the random variable \( Y = |X| \), where \( X \sim N(0, \sigma) \).
   a. Find the probability density function (PDF) of \( Y \).
   b. Find \( E(Y) \).
   
   \[ g(y) = \begin{cases} 0 & \text{if } y < 0 \\
   \frac{1}{\sqrt{2\pi \sigma^2}} e^{-y^2/(2\sigma^2)} & \text{if } y \geq 0
   \end{cases} \]
   
   \[ f(y) = \frac{1}{\sqrt{2\pi \sigma^2}} e^{-y^2/(2\sigma^2)} \]
   
   \[ E(Y) = \int_{0}^{\infty} y g(y) dy = \frac{\sigma}{\sqrt{\pi}} \]

3. (20%) Consider the random variable \( Y = X_1 + 2X_2 + X_3 + X_4 \)
   where \( X_1 \sim N(4, \sqrt{3}) \), \( X_2 \sim N(4, \sqrt{4}) \), \( X_3 \sim N(2, \sqrt{4}) \), and \( X_4 \sim N(3, \sqrt{2}) \).
   a. Find the mean of \( Y \).
   b. Find the variance of \( Y \).
   c. Find the probability that \( 15 \leq Y \leq 20 \).

We know that when we add independent random variables, their means add and their variances add.

\[ \mu_Y = 4 + 2(4) + 2 + 3 = 17 \]
\[ \sigma_Y^2 = 3 + 2^2(4) + 4 + 2 = 25 \]
\[ P(15 < Y < 20) = P\left( \frac{15-17}{\sqrt{25}} < z < \frac{20-17}{\sqrt{25}} \right) \]
\[ = P(-0.400 < z < 0.600) \]
\[ = 0.3812 \]