EECS 461 Short Quiz #3
Probability and Statistics
April 25, 2007

Name: KEY

Closed Book and Closed Notes: Show all work. Provide numerical answers as four-place demimals.

1. (40 %) A police crime lab suspects that the purity of *methamphetamine* in the local area has changed recently. In the past, the drug was on average 21.0 percent pure. In a random sample of twelve recent drug busts, the sample mean was 23.4 percent purity, with a sample standard deviation of 4.1 percent purity.

    a. Test the null hypothesis $H_o : \mu = 21$ against the alternate hypothesis $H_a : \mu \neq 21$ using $\alpha = 0.05$. Do you accept or reject the null hypothesis?

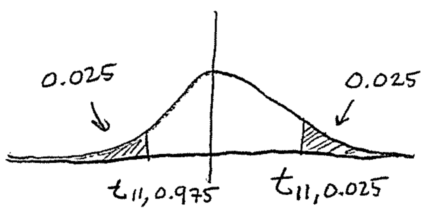
    b. This time, test the null hypothesis $H_o : \mu = 21$ against the alternate hypothesis $H_a : \mu > 21$ using $\alpha = 0.05$. Do you accept or reject the null hypothesis in this case?

    c. Calculate the p-value for the test in (a). State whether you accept or reject the null hypothesis in (a) by using the p-value.

    d. Calculate the p-value for the test in (b). State whether you accept or reject the null hypothesis in (b) by using the p-value.

$n=12$ $\quad \bar{X} = 23.4 \quad s^2 = (4.1)^2$, We do not know the true variance so we need to use the t distribution

a)



Two sided hypothesis

According to the table (and my calculator)

$t_{11,0.975} = -2.201$

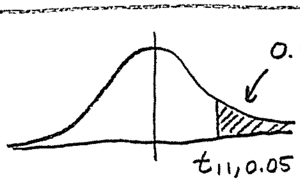
$t_{11,0.025} = 2.201$

the value we observe is $t_{11} = \dfrac{\bar{X}-\mu}{S/\sqrt{n}} = \dfrac{23.4-21.0}{4.1/\sqrt{12}} = \boxed{2.0278}$

This value is in between our two limits.

$t_{11,0.975} < t_{11} < t_{11,0.025}$ so $\boxed{\text{accept } H_0: \mu = 21}$

b)

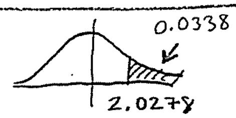


Now that we have a one sided hypothesis, we are looking for

$t_{11,0.05} = 1.796$

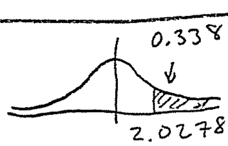
In this case the value we observe, $t_{11} = 2.0278$ exceeds the limit $t_{11} \nless t_{11,0.05}$ so $\boxed{\begin{array}{l}\text{Reject } H_0: \mu = 21 \\ \text{Accept } H_a: \mu > 21\end{array}}$

c)



The area under the curve to the right of $t_{11} = 2.0278$ is 0.0338, and this is a two sided hypothesis, so $p = 2 \times 0.0338 = 0.0675$, thus this test is not significant enough to overturn the null hypothesis in an $\alpha = 0.05$ test

d)



In a one sided hypothesis we have $p = 0.0338$, which <u>is</u> significant enough to overturn the null hypothesis in an $\alpha = 0.05$ test

2. (40 %) Consider the random variable $Y = |X|$, where $X \sim N(0, \sigma)$.

a. Find the probability density function (PDF) of Y.

b. Find $E(Y)$.

a) The random variable X has $f(x)$ and $F(x)$

The random variable Y has $g(x)$ and $G(y)$

this is a key step

$$G(y) = P(Y<y) = P(|X|<y) = \overbrace{P(-y < X < y)}^{\text{this is a key step}}$$

$$\boxed{= F(y) - F(-y)}$$

chain rule, like $\frac{d\cos(x^2)}{dx} = \sin(x^2)(2x)$

$$f(y) = \frac{dG(y)}{dy} = f(y) - f(-y)(-1)$$

$$= \frac{1}{\sqrt{2\pi\sigma^2}} e^{-y^2/2\sigma^2} + \frac{1}{\sqrt{2\pi\sigma^2}} e^{-y^2/2\sigma^2}$$

$$\boxed{= \sqrt{\frac{2}{\pi\sigma^2}}\, e^{-y^2/2\sigma^2}, \quad y \geq 0}$$

$$E(Y) = \int_0^\infty y \left(\sqrt{\frac{2}{\pi\sigma^2}}\, e^{-y^2/2\sigma^2}\right) dy$$

$$= \sqrt{\frac{2}{\pi\sigma^2}}(-\sigma^2)\int_0^\infty \left(-\frac{1}{\sigma^2}\right) y e^{-y^2/2\sigma^2} = \sqrt{\frac{2}{\pi\sigma^2}}(-\sigma^2)\, e^{-y^2/2\sigma^2}\Big|_0^\infty \boxed{= \sqrt{\frac{2\sigma^2}{\pi}}}$$

3. (20 %) Consider the random variable

$$Y = X_1 + 2X_2 + X_3 + X_4$$

where $X_1 \sim N(4, \sqrt{3})$, $X_2 \sim N(4, \sqrt{4})$, $X_3 \sim N(2, \sqrt{4})$, and $X_4 \sim N(3, \sqrt{2})$.

a. Find the mean of Y.

b. Find the variance of Y.

c. Find the probability that $15 \leq Y \leq 20$.

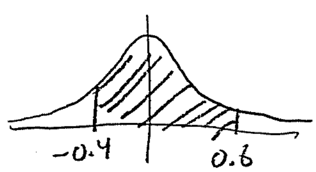
We know that when we add independent random variables their means add and their variances add

a) $\mu_Y = 4 + 2(4) + 2 + 3 \boxed{= 17}$

b) $\sigma_Y^2 = 3 + 2^2(4) + 4 + 2 \boxed{= 25}$

c) $P(15 < Y < 20) = P\left(\frac{15-17}{\sqrt{25}} < z < \frac{20-17}{\sqrt{25}}\right)$

$= P(-0.400 < z < 0.600)$



$\boxed{= 0.3812}$