Example #4

A soda bottling company uses a machine to fill the cans. If the variance of the fill volume exceeds 0.02 (fluid ounces)^2, then an unacceptably large number of cans will be underfilled. The bottler is interested in testing the hypothesis:

$$H_0: \sigma^2 = 0.02$$

$$H_a: \sigma^2 > 0.02 \quad (\text{a one-sided hypothesis})$$

In a random sample of $n=20$ cans the sample variance is

$$s^2 = 0.0225$$

Using a significance level of 5%, what can we conclude?

Use the χ^2_{n-1} random variable, the value we observed is

$$\chi^2_{19} = \frac{(n-1)s^2}{\sigma^2} = \frac{19(0.0225)}{0.02} = 21.375$$

The critical value in this instance is

$$\chi^2_{19, 0.05} = 30.1435$$

Since $21.35 < 30.14$, the evidence is not strong enough to indicate that the variance is greater than 0.02,

in fact, the p-value is 0.316, so instead of being 95% confident, in this case we are only 68.4% confident.