A soda bottling company uses a machine to fill the cans. If the variance of the fill volume exceeds 0.02 (fluid ounces)$^2$, then an unacceptably large number of cans will be underfilled. The bottler is interested in testing the hypothesis:

$$H_0: \sigma^2 = 0.02$$

$$H_a: \sigma^2 > 0.02 \quad (a \text{ one-sided hypothesis})$$

In a random sample of $n=20$ cans, the sample variance is $s^2 = 0.0225$.

Using a significance level of $5\%$, what can we conclude?

Use the $X^2_n$ random variable; the value we observed is

$$X^2_{19} = \frac{(n-1)s^2}{\sigma^2} = \frac{19 \times (0.0225)}{0.02} = 21.375$$

The critical value in this instance is

$$X^2_{19, 0.05} = 30.1435$$

Since $21.375 < 30.14$, the evidence is not strong enough to indicate that the variance is greater than 0.02, in fact, the $p$-value is 0.316, so instead of being $95\%$ confident, in this case we are only $68.4\%$ confident.