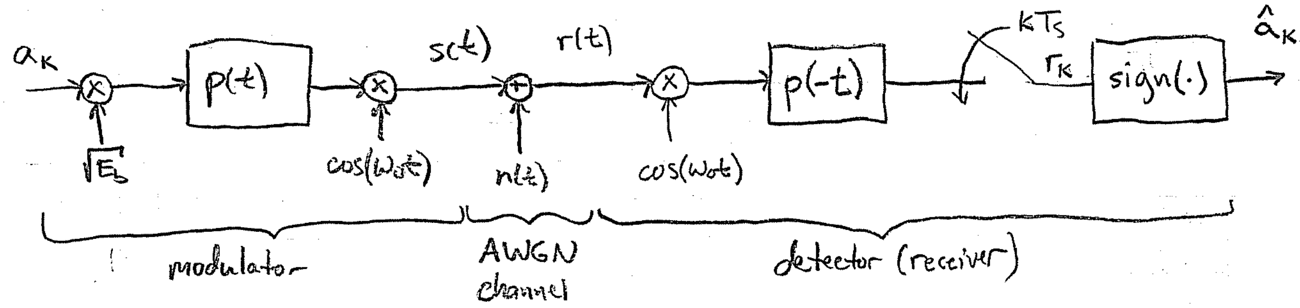
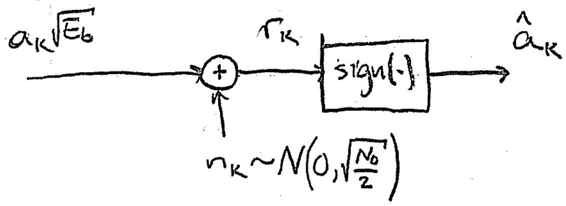
Probability of Bit Error for BPSK (Binary Phase Shift Keying)

A BPSK system modulates antipodal bits, $a_K \in \{\pm 1\}$, and transmits them over a channel. We will assume the additive white Gaussian noise (AWGN) channel.



- $p(t)$ is chosen to shape the bandwidth (spectrum) of the signal
- $n(t)$ is a Gaussian random process with zero mean and power spectral density $\frac{N_0}{2}$
- We will use a simplified discrete-time model



$$r_K = a_K\sqrt{E_b} + n_K$$

- n_K is a Gaussian (normal) RV with $\mu = 0$ and $\sigma^2 = \frac{N_0}{2}$

Let E be the event that $\hat{a}_K \neq a_K$

Use Baye's Rule

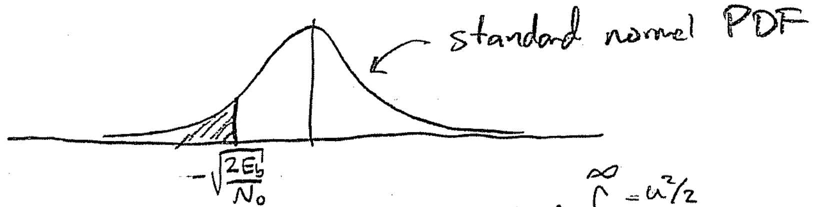
$$P(E) = P(E \mid a_K = +1)P(a_K = +1) + P(E \mid a_K = -1)\,P(a_K = -1)$$

Computation of $P(E \mid a_K = +1)$

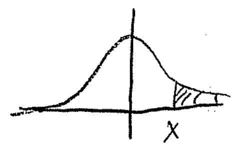
If we get an error when we know $a_K = +1$, then it is because r_K was negative

$$P(E \mid a_K = +1) = P(r_K < 0 \mid a_K = +1) = P\left(\sqrt{E_b} + n_K < 0\right)$$

$$= P\left(n_K < -\sqrt{E_b}\right) = P\left(\frac{n_K}{\sqrt{\frac{N_0}{2}}} < -\sqrt{\frac{2E_b}{N_0}}\right) = P\left(z < -\sqrt{\frac{2E_b}{N_0}}\right)$$



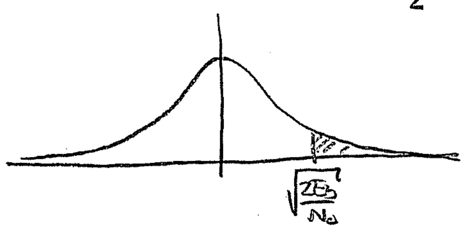
In communications, we define $Q(x) \triangleq \int_{x}^{\infty} e^{-u^2/2}\, du$



Therefore $P(E \mid a_K = +1) = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$

Computation of $P(E \mid a_K = -1)$

$$P(E \mid a_K = -1) = P\left(-\sqrt{E_b} + n_K > 0\right) = P\left(\frac{n_K}{\sqrt{\frac{N_0}{2}}} > \sqrt{\frac{2E_b}{N_0}}\right) = P\left(z > \sqrt{\frac{2E_b}{N_0}}\right) = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$



If we assume that the bits are equally likely to be transmitted, i.e. $P(a_K = +1) = P(a_K = -1)$, then

$$\boxed{P(E) = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)}$$

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