

The lifetime of a lightbulb is an exponentially distributed random variable T , where T is measured in hours and the expected lifetime is 800 hours. You consider a "duel" to be a bulb that lasts less than 400 hours.

You have a box of six bulbs, what is the probability that we have to wait until the 6th bulb to get a duel

In Chapter 2 we dealt a lot with Bernoulli trials. We can easily identify Bernoulli trials by scanning the problem and looking for something that can be modeled by a "loaded coin toss." Even though we are in Chapter 3, we should be able to notice that the "Duel"/"no Duel" classification can be modeled as a loaded coin toss. We are being asked to find the waiting time for a successful outcome of a Bernoulli trial.

So we are looking for $P(X=6)$ for a geometric random variable. The new twist in this problem is that now we have the information we need to compute p (In the past, p was usually given to us)

$$P(\text{lightbulb is a "duel"}) = p = P(0 \leq T \leq 400) = F(400)$$

↙ exponential RV

$$= 1 - e^{-400/800}$$

$= 0.3935$

↙ geometric RV

$$P(X=6) = q^5 p = (0.6065)^5 (0.3935)$$

$= 0.0323$