The number of times that an individual contracts a cold in a given year is a Poisson random variable with parameter $\lambda = 3$. Suppose a new wonder drug (based on large quantities of vitamin C) has just been marketed that reduces the Poisson parameter to $\lambda = 2$ for 75 percent of the population. For the other 25 percent of the population, the drug has no appreciable effect on colds. If an individual tries the drug for a year and has 0 colds in that time, how likely is it that the drug is beneficial for him or her?

OK, let's define

A_1 : event that the drug was beneficial for the individual
A_2 : " " " " " not beneficial " " "
(Partition)

B : The individual had 0 colds in the year

We are looking for

$$P(A_1|B)$$

This might not have jumped out at you, but this is what the last sentence says.

Apply Bayes' Rule

$$P(A_1|B) = \frac{P(B|A_1)P(A_1)}{P(B|A_1)P(A_1) + P(B|A_2)P(A_2)} = \frac{0.1353 \times 0.75}{0.1353 \times 0.75 + 0.0499 \times 0.25}$$

$$\boxed{= 0.8908}$$

$P(A_1) = 0.75$

$P(A_2) = 0.25$

Now, what about $P(B|A_i)$. Well, the colds arrive whenever they want, we can count the colds, but we cant count the "not colds", all we know is the expected number of colds (λ), and A_1 and A_2 tell us λ.

$$P(B|A_1) = P(0 \le X \le 0) = \sum_{x=0}^{0} e^{-2} \frac{2^x}{x!} = 0.1353$$

$$P(B|A_2) = P(0 \le X \le 0) = \sum_{x=0}^{0} e^{-3} \frac{3^x}{x!} = 0.0499$$