

5-37. A mathematics textbook has 200 pages on which typographical errors in the equations could occur. If there are in fact five errors randomly dispersed among these 200 pages, what is the probability that a random sample of 50 pages will contain at least one error? How large must the random sample be to assure that at least three errors will be found with 90% probability?

There is a big book with a certain number of errors, and we are taking a smaller sample and are wondering how many errors we will get. This sound like a hypergeometric RV.

let A be the event that there is at least one error in our sample of 50. X is hypergeometric with $N=200$, $n=50$

and $D=5$

$$P(A) = P(1 \leq X \leq 5) = \sum_{x=1}^5 \frac{\binom{5}{x} \binom{195}{50-x}}{\binom{200}{50}} = 0.7667$$

Now we are asked to adjust the size of the sample, n , until we have a 90% chance of catching 3 or more errors

Find n such that

$$\sum_{x=3}^5 \frac{\binom{5}{x} \binom{95}{n-x}}{\binom{200}{n}} > 0.90$$

By process of trial and error, this is satisfied with

$$n = 151$$

Can also use binomial approximation to the hypergeometric distribution when solving this problem. $P(X \geq k) \approx \sum_{x=k}^n \binom{n}{x} p^x (1-p)^{n-x}$