5-21. Three companies, X, Y, and Z, have probabilities of obtaining an order for a particular type of merchandise of 0.4, 0.3, and 0.3, respectively. Three orders are to be awarded independently. What is the probability that one company receives all the orders?

Define event A = one company receives all 3 orders

This can occur in 3 ME ways

$$A = A_x \cup A_y \cup A_z$$

where A_X = company X receives all 3 orders

A_y = " Y " " " "

A_z = " Z " " " "

$$P(A) = P(A_x \cup A_y \cup A_z) = \overbrace{P(A_X) + P(A_y) + P(A_z)}^{\text{since they are ME}}$$

From Company X's point of view orders come in (Success) or don't come in (Failure) with probabilities 0.4 and 0.6, respectively. Event A_X occurs if the negative binomial RV X, with $r=3$ and $p=0.3$, assumes values in the range $3 \le X \le 3$. In other words, we need all 3 successes without any failures. Similar arguments hold for the other two companies.

$$P(A_x) = P(3 \le X \le 3) = \sum_{x=3}^{3} \binom{2}{2} (0.4)^3 (0.6)^{x-3} = (0.4)^3$$

$$P(A_y) = P(3 \le Y \le 3) = \sum_{y=3}^{3} \binom{2}{2} (0.3)^3 (0.7)^{y-3} = (0.3)^3$$

$$P(A_z) = P(3 \le Z \le 3) = \sum_{z=3}^{3} \binom{2}{2} (0.3)^3 (0.7)^{z-3} = (0.3)^3$$

$$P(A) = P(A_X) + P(A_y) + P(A_z) = (0.4)^3 + 2(0.3)^3 \boxed{= 0.118}$$