A communications channel transmits the digits 0 and 1. However, due to static, the digit transmitted is incorrectly received with probability .2. Suppose that we want to transmit an important message consisting of one binary digit. To reduce the chance of error, we transmit 00000 instead of 0 and 11111 instead of 1. If the receiver of the message uses "majority" decoding, what is the probability that the message will be incorrectly decoded? What independence assumptions are you making? (By majority decoding we mean that the message is decoded as "0" if there are at least three zeros in the message received and as "1" otherwise.)

Let E be the event that we are incorrect. We are looking for $P(E)$

Let the RV X count the number of binary digits that are in error. Clearly, if the individual errors are independent, which we will assume, then X is a binomial RV with $n=5$ and $p=0.2$.

$$P(E) = P(3 \le X \le 5)$$

$$= \sum_{x=3}^{5} \binom{5}{x} 0.2^{x}\, 0.8^{5-x}$$

$$\boxed{= 0.0579}$$

This is a good approach since we have reduced the probability of error from 0.2 to only 0.0579. But, we are consuming more bandwidth since we have to transmit 5 bits for every one information bit. This is an example of an error control code that can correct up to 2 bit errors at the receiver.