

2-10. A random variable  $X$  has a CDF of the form

$$F_X(x) = 1 - \left(\frac{1}{2}\right)^{x+1}, \quad x = 0, 1, 2, \dots,$$
$$= 0, \quad x < 0.$$

- (a) Find the probability function for  $X$ .  
(b) Find  $P_X(0 < X \leq 8)$ .

Notation: We are given the CDF  $F(x) = F_X(x)$

a) find the PDF

$$P(X=x) = f(x) = F(x) - F(x-1)$$

$$= 1 - \left(\frac{1}{2}\right)^{x+1} - \left[1 - \left(\frac{1}{2}\right)^x\right], \quad x = 1, 2, \dots$$

$$= \left(\frac{1}{2}\right)^x - \left(\frac{1}{2}\right)^{x+1}$$

$$= \left(\frac{1}{2}\right)^x \left[1 - \frac{1}{2}\right]$$

$$= \left(\frac{1}{2}\right)^x \frac{1}{2}$$

$$= \left(\frac{1}{2}\right)^{x+1} \quad x = 1, 2, \dots$$

What about  $P(X=0)$ ?

$$P(X=0) = F(0) - \overset{\text{given as zero}}{F(-1)} = \frac{1}{2}$$

$$P(X=x) = f(x) = \left(\frac{1}{2}\right)^{x+1}, \quad x = 0, 1, \dots$$

Exercise for the student:  
Does this sum to 1

$$b) \quad P(0 \leq X \leq 8) = \sum_{x=0}^8 \left(\frac{1}{2}\right)^{x+1} = 0.9980$$