

Example using Expectation

Suppose X is a random variable such that $E(X) = 3 = \mu_X$
and $\text{Var}(X) = 5 = \sigma_X^2$.

Let H be a new random variable that can be
written in terms of X

$$H(X) = 2X - 7.$$

We are asked to find the mean and variance of H . We
have 2 options

① Obtain the PDF of H

② Use the PDF of X

Let's do option ② since we know hardly anything
about H , and we know a little more about X

$$\begin{aligned}\mu_H &= E(H) = E(2X - 7) = E(2X) - E(7) \leftarrow \text{expectation is linear} \\ &= 2E(X) - 7 \leftarrow \text{expected value of a constant is the constant} \\ &= 6 - 7 \\ &= -1\end{aligned}$$

$$\begin{aligned}\sigma_H^2 &= E((H - \mu_H)^2) = E((2X - 7 + 1)^2) = E(4X^2 - 24X + 36) \\ &= 4E(X^2) - 24E(X) + 36 \\ &= 4(\underbrace{\sigma_X^2 + \mu_X^2}) - 36 \\ &= 4(5 + 9) \\ &= 20\end{aligned}$$