EECS 360 Short Quiz #3

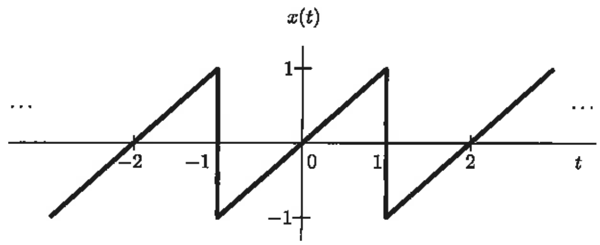
Signal and System Analysis

March 6, 2012

Name: KEY

Open book, open notes, no calculator. Be neat, write legibly. For full credit you must show all work and justify each answer. You may write on the both sides of the paper, and use additional sheets of paper if needed.

1. (40 %) Find the CTFS harmonic function $X[k]$ for the periodic signal below.



$\boxed{\begin{array}{l}\text{Let } T_F = T_0 = 2 \\ f_F = 1/2\end{array}}$

$$X[k] = \frac{1}{T_F}\int_{T_F} x(t)\, e^{-j2\pi k f_F t}\, dt$$

↑ I choose to integrate from −1 to +1, which is $T_F = 2$ seconds long

$$= \frac{1}{2}\int_{-1}^{1} t\, e^{-j\pi k t}\, dt$$

Integration by parts $\quad u = t \quad dv = e^{-j\pi k t}$

$\quad du = 1 \quad v = \frac{1}{-j\pi k} e^{-j\pi k t}$

$$= \left[\frac{t}{-j2\pi k} e^{-j\pi k t}\right]_{t=-1}^{1} - \frac{1}{-j2\pi k}\int_{-1}^{1} e^{-jk\pi t}\, dt$$

$$= \frac{1}{-j2\pi k} e^{-j\pi k} - \frac{-1}{-j2\pi k} e^{j\pi k} - \left[\frac{1}{-2(\pi k)^2} e^{-jk\pi t}\right]_{t=-1}^{1}$$

$$= \frac{\cos(\overset{(-1)^k}{\cancel{\pi k}})}{-j\pi k} + \frac{\sin(\overset{0}{\cancel{\pi k}})}{j(\pi k)^2}$$

← $\boxed{k = 0, \text{ clearly } x(t) \text{ has no DC term}}$

$$\boxed{= j\frac{(-1)^k}{\pi k}}$$

2. (60 %) A periodic signal with fundamental period T_0 is said to be *half-wave symmetric* if it satisfies the relationship

$$x(t) = -x(t - T_0/2) \tag{1}$$

In words, a half-wave symmetric signal has one half of its period that is exactly the negative of the other half; $\sin(t)$ and $\cos(t)$ are both half-wave symmetric. In this problem, we will show that $X[2k] = 0$ for all half-wave symmetric signals (i.e., the even harmonics of the CTFS are zero).

(a) Let $x_1(t) \longleftrightarrow X_1[k]$ and $x_2(t) \longleftrightarrow X_2[k]$ be CTFS pairs, and let $x_2(t) = -x_1(t - T_0/2)$. Because $x_2(t)$ and $x_1(t)$ are related, use the CTFS properties to express $X_2[k]$ in terms of $X_1[k]$.

(b) If $x_1(t)$ is half-wave symmetric, then Equation (1) tells us that $x_2(t) = x_1(t)$, and therefore

$$X_2[k] = X_1[k] = \text{[the answer you got in part (a)]}.$$

Based on these facts, show that the even harmonics of the CTFS, $X_1[2k]$, must be zero.

(c) Does a half-wave symmetric signal have a DC offset? Why or why not?

(a) $x_2(t) = -\underbrace{x_1(t - T_0/2)}_{\text{use time shift property}} \longleftrightarrow \boxed{X_2[k] = -X_1[k]e^{-j\pi k}}$

(b) $X_2[k] = X_1[k] = -X_1[k]\underbrace{e^{-j\pi k}}_{\text{for } 2k \text{ we have } e^{j2\pi k} = 1}$

therefore, for $2k$ we have $\underbrace{X_1[2k] = -X_1[2k]}_{\text{there is only one number that is equal to its negative} \Rightarrow \text{zero}}$

$\boxed{\Rightarrow X_1[2k] = 0}$

(c) Because $X_1[2k] = 0$

then $\boxed{X_1[0] = 0}$

$\Rightarrow$ no DC Offset