EECS 360 Short Quiz #2
Signal and System Analysis
February 21, 2012

Name: KEY

Open book, open notes, no calculator. Be neat, write legibly. For full credit you must show all work and justify each answer. You may write on the both sides of the paper, and use additional sheets of paper if needed.

1. (25 %) When the unit step function, $u(t)$, is applied as the input to an LTI system, the output produced by the system is $s(t) = (1 - e^{-t})u(t)$. Determine the impulse response of the system, $h(t)$.

In class, we learned that the step response and the impulse response are related ($s(t) = \int_{-\infty}^{t} h(\tau)d\tau$, $h(t) = \frac{d}{dt}s(t)$)

This is pretty easy to believe because of the relationship between $\delta(t)$ and $u(t)$ ($u(t) = \int_{-\infty}^{t} \delta(\tau)d\tau$, $\delta(t) = \frac{d}{dt}u(t)$)

Therefore

$$h(t) = \frac{d}{dt}s(t)$$

product rule $g'(t)h(t) + g(t)h'(t)$

$$= \frac{d}{dt}\left[(1-e^{-t})u(t)\right]$$

$$= \frac{d}{dt}\left[1-e^{-t}\right]u(t) + (1-e^{-t})\frac{d}{dt}\left[u(t)\right]$$

$$= e^{-t}u(t) + \underbrace{\underbrace{(1-e^{-t})}_{\text{evaluate at } t=0}\delta(t)}_{\text{sampling property}}$$

this is "on" only at $t=0$

$$= e^{-t}u(t) + (1-e^{0})\delta(t)$$

$$\boxed{= e^{-t}u(t)}$$

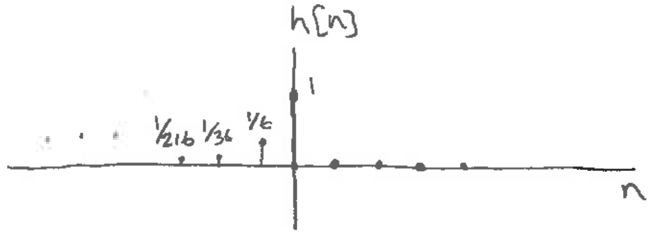
2. (35 %) For each of the following impulse responses, determine if the system is (i) memoryless, (ii) causal, and (iii) stable:

(a) $h[n] = 6^n u[-n]$

(b) $h(t) = e^{-5t}\sin(2\pi t)u(t)$

(c) $h[n] = \sum_{m=-\infty}^{\infty} (-1)^m \delta[n - 2m]$

(a)

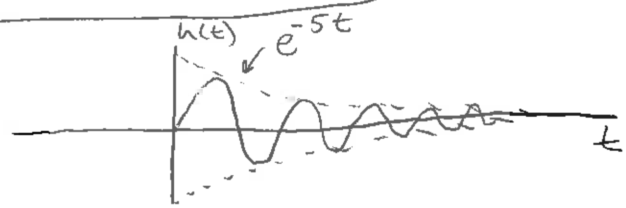


(i) it has memory because it is not $h[n] = A\delta[n]$

(ii) it is the exact opposite of causal (anticausal) $h[n] = 0 \quad n > 0$

(iii) $\sum_{n=-\infty}^{\infty} |h[n]| = 6/5 < \infty$, yes it is stable

(b)

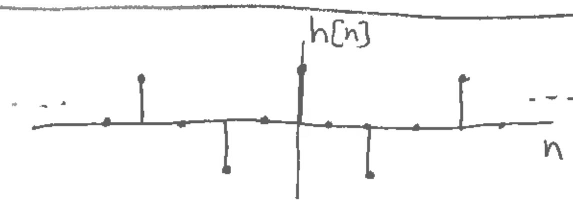


(i) it has memory because it is not $h(t) = A\delta(t)$

(ii) it is causal because $h(t) = 0$ for $t < 0$

(iii) $\int_{-\infty}^{\infty} |h(t)|dt =$ I don't know, but $< \int_{-\infty}^{\infty} e^{-5t} dt = \frac{1}{5} < \infty$ stable

(c)



(i) has memory because $h[n]$ is not $A\delta[n]$

(ii) it is not causal because $h[n] \neq 0$ for $n < 0$

(iii) $\sum_{n=-\infty}^{\infty} |h[n]| = \cdots + 1 + 1 + 1 + \cdots = \infty$, it is unstable

3. (40 %) The input to an LTI system is $x(t) = 1.5\text{rect}((t - 0.5)/5)$. The impulse response of this LTI system is $h(t) = 2\text{rect}((t - 0.5)/3)$.

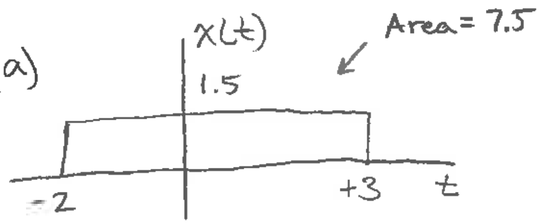
(a) Graph $x(t)$ and $h(t)$.

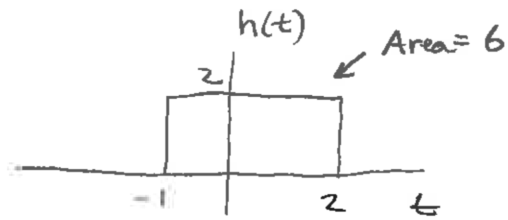
(b) At what time does the output, ~~$h(t)$~~ $y(t)$, "turn on"?

(c) At what time does the output, ~~$h(t)$~~ $y(t)$, "turn off"?

(d) What is the output, $y(t)$? [You may give your answer as a graph, or as a function, whichever you prefer.]

(a)





(b) $t_{y_0} = t_{x_0} + t_{h_0} = -2 - 1 \boxed{= -3}$

(c) $t_{y_1} = t_{x_1} + t_{h_1} = 3 + 2 \boxed{= 5}$

Double check

$\Delta t_y = \Delta t_x + \Delta t_h$
$= 5 + 3 = 8$

(d) <u>Facts about y(t)</u>

- starts at $t = -3$, stops at $t = 5$
- the two curves are flat lines (constant) and the integral of a constant is a sloped line and in this case the slope will be $1.5 \cdot 2 = 3$
- The maximum value of $y(t)$ will be when the narrower rect [$h(t)$] is completely inside the wider rect [$x(t)$] and this will occur for a duration of 2 seconds (the difference in their widths)
- This maximum value will be $1.5 \cdot$ (area of $h(t)$) $= 9$ — amplitude of $x(t)$
- Area of y = (Area of x) $\cdot$ (Area of h) $= 7.5 \cdot 6 = 45$