EECS 360 Short Quiz #5

Signal and System Analysis

May 3, 2011

Name: KEY

Open book, open notes, no calculator. Be neat, write legibly. For full credit you must show all work and justify each answer. You may write on the both sides of the paper, and use additional sheets of paper if needed.

1. (35 %) The continuous-time signal $x(t) = \sin(200\pi t) + 2\cos(150\pi t)$ is sampled with an impulse train $\delta_{T_s}(t)$, which results in the signal $x_\delta(t)$.

$\boxed{f_{max} = 100\text{Hz}}$

(a) Sketch the CTFT of $x_\delta(t)$, i.e. $X_\delta(f)$, for the following sample rates:

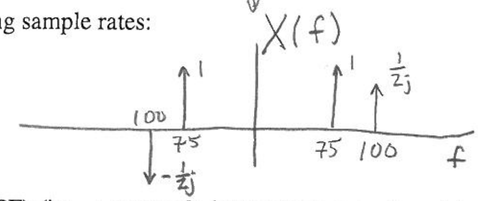
to avoid aliasing, the sampling rate must be greater than $2 \cdot 100 = 200$Hz

i. $f_s = 100$ samples/s ← aliasing

ii. $f_s = 200$ samples/s ← aliasing

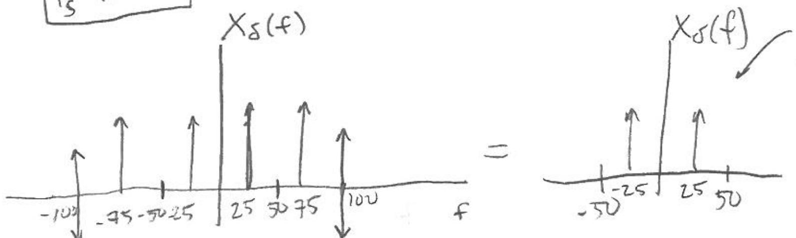
iii. $f_s = 400$ samples/s ← no aliasing

iv. $f_s = 500$ samples/s ← no aliasing



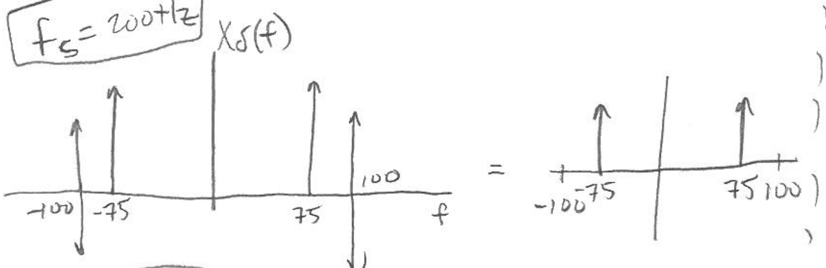
(b) The signal $x_\delta(t)$ is fed to an ideal low-pass filter (LPF) (i.e., a rectangle in the frequency domain) with a cut-off frequency of $f_c = f_s/2$. The output of the LPF is $y(t)$, which is sometimes called the *reconstructed signal*. For each of the sample rates in part (a), determine the LPF output, $y(t)$.

$\boxed{f_s = 100\text{Hz}}$



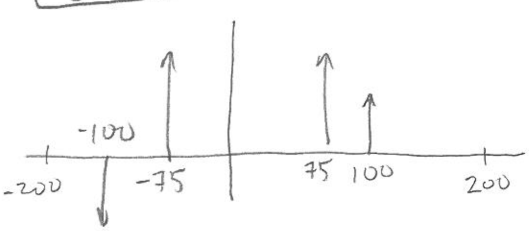
→ LPF $f_c = 50$Hz → $y(t) = 2\cos(50\pi t)$

$\boxed{f_s = 200\text{Hz}}$



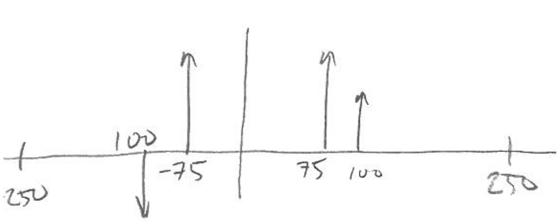
→ LPF $f_c = 100$Hz → $y(t) = 2\cos(150\pi t)$

$\boxed{f_s = 400\text{Hz}}$



→ LPF $f_c = 200$Hz → $y(t) = x(t)$
$= \sin(200\pi t) + 2\cos(150\pi t)$

$\boxed{f_s = 500\text{Hz}}$



→ LPF $f_c = 250$Hz → $y(t) = x(t)$
$= \sin(200\pi t) + 2\cos(150\pi t)$

← (a) (b) →

2. (35 %) A linear time-invariant system has an impulse response $h(t)$ and a Laplace transform

$$H(s) = K\frac{1}{(s-p_0)(s-p_1)(s-p_2)(s-p_3)}$$

which satisfies the following properties: duh.

(a) $H(s)$ has four poles but no zeros.

(b) $h(t)$ is even and real-valued.

(c) $H(s)$ has a complex-valued pole at $s = 0.5e^{j\pi/4}$.

(d) the area enclosed by the impulse response is 8, i.e. $\int_{-\infty}^{\infty} h(t)\,dt = 8$.

Determine the Laplace transfer function $H(s)$ and the associated ROC.

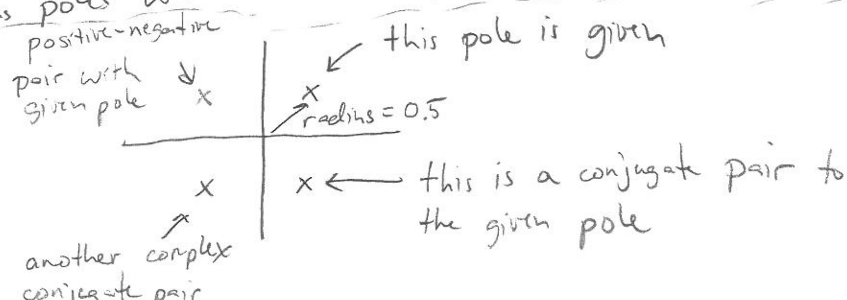
(b) Because $h(t)$ is real-valued, its poles are either real and distinct, or they come in complex-conjugate pairs

Because $h(t)$ is even, its right-sided part is $h_R(t) = h(t)u(t)$

left-sided part is a mirror image $h_L(t) = h_R(-t)$

Comparing entries in the Laplace transform table, such a function has poles where the real parts are positive-negative pairs

(c)

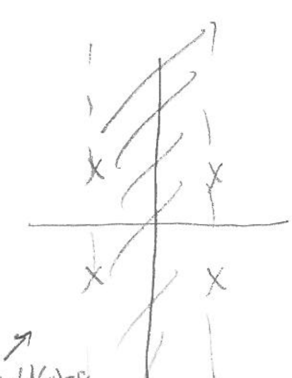


$$H(s) = K\frac{1}{(s-0.5e^{+j\pi/4})(s-0.5e^{-j\pi/4})(s-0.5e^{+j3\pi/4})(s-0.5e^{-j3\pi/4})}$$

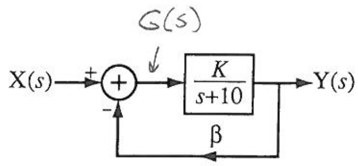
$$= \boxed{K\frac{1}{\left(s^2-\frac{1}{\sqrt{2}}s+\frac{1}{4}\right)\left(s^2+\frac{1}{\sqrt{2}}s+\frac{1}{4}\right)}}$$

(d) $$\int_{-\infty}^{\infty} h(t)\,dt = \int_{-\infty}^{\infty} h(t)e^{-st}\,dt\Big|_{s=0} = H(s)\Big|_{s=0} = 8$$

$$H(0) = K\frac{1}{(0-0+\frac{1}{4})(0+0-\frac{1}{4})} = 16K = 8 \implies \boxed{K = \frac{1}{2}}$$



because $H(0)=8$, we know ROC includes $H(0)$



3. (30 %) Find the expression for the overall system transfer function of the system in the figure above. Answer the following questions about the transfer function you have found:

   (a) Let $\beta = 1$. For what values of K is the system stable?

   (b) Let $\beta = -1$. For what values of K is the system stable?

   (c) Let $\beta = 10$. For what values of K is the system stable?

$$G(s) = X(s) - \beta Y(s)$$

$$Y(s) = \frac{K}{s+10} G(s) = \frac{K}{s+10}\left[X(s) - \beta Y(s)\right]$$

$$= \frac{K}{s+10} X(s) - \frac{K\beta}{s+10} Y(s)$$

$$\Rightarrow \quad Y(s)\left[1 + \frac{K\beta}{s+10}\right] = \frac{K}{s+10} X(s)$$

$$\Rightarrow \quad \frac{Y(s)}{X(s)} = H(s) = \frac{K}{\cancel{s+10}}\left[\frac{\cancel{s+10}}{s+10+K\beta}\right] = \boxed{\frac{K}{s+10+K\beta}}$$

(a) To be stable, the real part of the pole must be negative (<0)

The pole is at $-10 - K\beta$

Let $\beta = +1 \quad \Rightarrow \quad -10 - K < 0 \quad \Rightarrow \quad \boxed{-10 < K}$

(b) $-10 + K < 0 \quad \Rightarrow \quad \boxed{K < 10}$

(c) $-10 - 10K < 0 \quad \Rightarrow \quad \boxed{-1 < K}$