EECS 360 Short Quizzes #3 & #4

Signal and System Analysis
April 7, 2011

Name: KEY

Open book, open notes, no calculator. Be neat, write legibly. For full credit you must show all work and justify each answer. You may write on the both sides of the paper, and use additional sheets of paper if needed.

1. (25 %) Consider the periodic signal $x(t)$, which has the Fourier series representation

$$x(t) = \sum_{k=-\infty}^{\infty} X[k] e^{j2\pi k f_F t}$$

The periodic signal $y(t)$ is related to $x(t)$ and has its own Fourier series representation

$$y(t) = \sum_{k=-\infty}^{\infty} Y[k] e^{j2\pi k f_F t}$$

Derive the relationship between $Y[k]$ and $X[k]$ when the relationships between $y(t)$ and $x(t)$ are given by:

(a) $y(t) = \frac{d}{dt} x(t)$
(b) $y(t) = x(t - t_0)$
(c) $y(t) = K x(t)$

(a) $$y(t) = \frac{d}{dt}\left[\sum_{k=-\infty}^{\infty} X[k] e^{j2\pi k f_F t}\right] = \sum_{k=-\infty}^{\infty} X[k] \frac{d}{dt}\left[e^{j2\pi k f_F t}\right] = \sum_{k=-\infty}^{\infty} \underbrace{j2\pi k f_F X[k]} e^{j2\pi k f_F t}$$

$$= \sum_{k=-\infty}^{\infty} Y[k] e^{j2\pi k f_F t}$$

therefore $\boxed{Y[k] = j2\pi k f_F X[k]}$

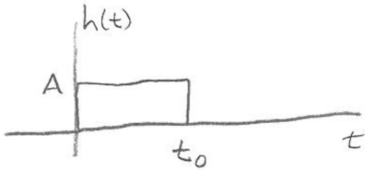
(b) $$y(t) = x(t-t_0) = \sum_{k=-\infty}^{\infty} X[k] e^{j2\pi k f_F (t-t_0)} = \sum_{k=-\infty}^{\infty} \underbrace{e^{-j2\pi k f_F t_0} X[k]} e^{j2\pi k f_F t} = \sum_{k=-\infty}^{\infty} Y[k] e^{j2\pi k f_F t}$$

therefore $\boxed{Y[k] = e^{-j2\pi k f_F t_0} X[k]}$

(c) $$y(t) = K x(t) = K \sum_{k=-\infty}^{\infty} X[k] e^{j2\pi k f_F t} = \sum_{k=-\infty}^{\infty} \underbrace{K X[k]} e^{j2\pi k f_F t} = \sum_{k=-\infty}^{\infty} Y[k] e^{j2\pi k f_F t}$$

therefore $\boxed{Y[k] = K X[k]}$

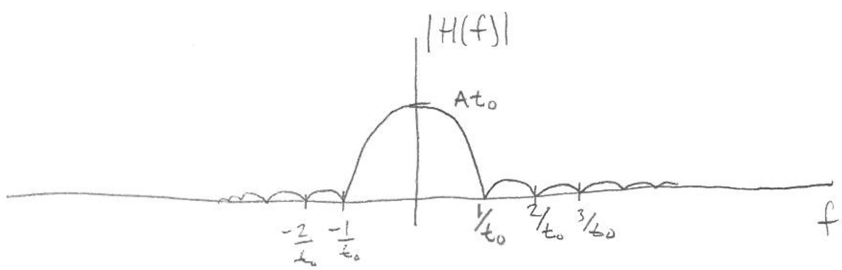
2. (20 %) One major problem in real instrumentation systems is electromagnetic interference caused by the 60 Hz power lines. A system with an impulse response of the form $h(t) = A(u(t) - u(t - t_0))$ can reject 60 Hz and all its harmonics. Find the numerical value of t_0 that makes this happen.



$$h(t) = A(u(t) - u(t - t_0)) = A\,\mathrm{rect}\left(\frac{t - t_0/2}{t_0}\right)$$

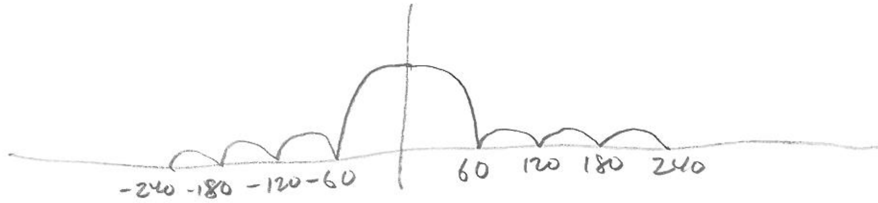
$$\Rightarrow H(f) = A t_0 e^{-j2\pi f t_0/2}\,\mathrm{rect}(t_0 f)$$

$$|H(f)| = |\mathrm{rect}(t_0 f)|$$ ← sinc is zero when its argument is an integer



we want to reject 60 Hz and all of its harmonics

so we want $|H(f)|$ to look like this



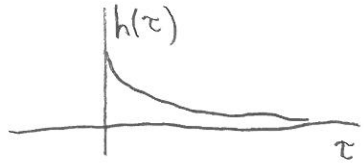
therefore $\frac{1}{t_0} = 60 \Rightarrow t_0 = \frac{1}{60}$

3. (25 %) Suppose the continuous-time signal $x(t) = e^{-t}u(t)$ is applied as input to a causal LTI system modeled by the impulse response $h(t) = e^{-2t}u(t)$. Calculate the resulting output $y(t)$ using:

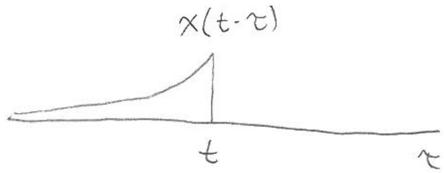
(a) direct convolution

(b) the properties of the CTFT

(a) We already did this in a HW problem



when $t < 0$ there is no overlap



when $t \geq 0$ we have

$$y(t) = \int_0^t e^{-2\tau} e^{-t} e^{\tau} \, d\tau$$

$$= e^{-t} \int_0^t e^{-\tau} \, d\tau$$

$$= -e^{-t}\left[e^{-t} - 1\right]$$

$$= e^{-t} - e^{-2t}$$

$$\Rightarrow \boxed{y(t) = \begin{cases} 0 & t < 0 \\ e^{-t} - e^{-2t} & t \geq 0 \end{cases}} = e^{-t}u(t) - e^{-2t}u(t)$$

(b) $y(t) = x(t) * h(t) \overset{\mathcal{F}}{\longleftrightarrow} Y(f) = X(f) \cdot H(f)$

$$X(f) = \frac{1}{1 + j2\pi f}$$

$$H(f) = \frac{1}{2 + j2\pi f}$$

$$Y(f) = \left(\frac{1}{1 + j2\pi f}\right)\left(\frac{1}{2 + j2\pi f}\right)$$

$$= \frac{A}{1 + j2\pi f} + \frac{B}{2 + j2\pi f}$$

$$= \frac{2A + B + (A+B)j2\pi f}{(1 + j2\pi f)(2 + j2\pi f)} \Rightarrow \begin{matrix} 2A + B = 1 \\ A + B = 0 \end{matrix} \Rightarrow \begin{matrix} A = 1 \\ B = -1 \end{matrix}$$

$$\boxed{y(t) = e^{-t}u(t) - e^{-2t}u(t)} \xleftarrow{\mathcal{F}} = \frac{1}{1 + j2\pi f} - \frac{1}{2 + j2\pi f}$$

4. (30 %) Consider the moving average systems

$$y_1[n] = \frac{1}{2}(x[n] + x[n-1]) \qquad \text{and} \qquad y_2[n] = \frac{1}{2}(x[n] - x[n-1])$$

The first system averages successive inputs, while the second forms the difference. The impulse responses are

$$h_1[n] = \frac{1}{2}(\delta[n] + \delta[n-1]) \qquad \text{and} \qquad h_2[n] = \frac{1}{2}(\delta[n] - \delta[n-1])$$

Find $H_1(F)$ and $H_2(F)$ [or $H_1(e^{j\Omega})$ and $H_2(e^{j\Omega})$] if you prefer] and plot their magnitudes. Classify the two systems as lowpass or highpass. [Recall that a lowpass filter blocks high-frequency components and a highpass filter blocks low-frequency components.]

(a)

$$h_1[n] = \frac{1}{2}(\delta[n] + \delta[n-1]) \xleftrightarrow{\mathcal{F}} H_1(F) = \frac{1}{2}\left(1 + e^{-j2\pi F}\right)$$

factor out half of this phase

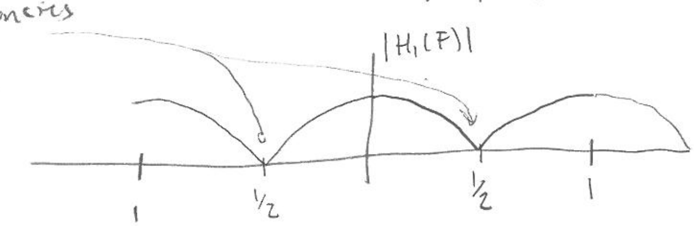
$$= \frac{1}{2} e^{-j\pi F}\left(e^{j\pi F} + e^{j\pi F}\right)$$

$$= e^{j\pi F} \cos(\pi F)$$

$$\Rightarrow |H_1(F)| = |\cos(\pi F)|$$

this system rejects high frequencies

$\boxed{\text{LOWPASS}}$



(b)

$$h_2[n] = \frac{1}{2}(\delta[n] - \delta[n-1]) \xleftrightarrow{\mathcal{F}} \frac{1}{2}\left(1 - e^{-j2\pi F}\right) = H_2(F)$$

$$= \frac{1}{2} e^{-j\pi F}\left(e^{j\pi F} - e^{-j\pi F}\right)$$

$$= j e^{-j\pi F} \sin(\pi F)$$

$$\Rightarrow |H_2(F)| = |\sin(\pi F)|$$

this system rejects low frequencies

$\boxed{\text{HIGH PASS}}$