EECS 360 Short Quiz #2

Signal and System Analysis

March 10, 2011

Name: KEY

Open book, open notes, no calculator. Be neat, write legibly. For full credit you must show all work and justify each answer. You may write on the both sides of the paper, and use additional sheets of paper if needed.

1. (35 %) Determine if systems with the following impulse responses are memoryless, causal, and stable. Justify your answers.

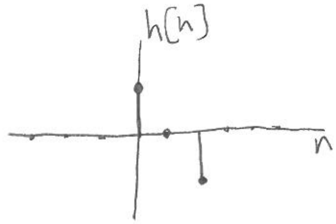
   (a) $h[n] = \delta[n] - \delta[n-2]$

   (b) $h(t) = 2\mathrm{rect}(t/2)$

   (c) $h[n] = 20(\frac{1}{4})^n u[n]$

   (d) $h(t) = [1 - e^{-4t}]u(t)$

(a)

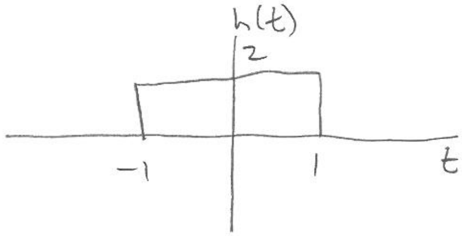


Because $h[n] = 0$ for $n < 0$ it is causal

Because $\sum_{n=-\infty}^{\infty} |h[n]| = 2 < \infty$ it is stable

Because $h[n] \neq 0$ for $n \neq 0$ it has memory

(b)

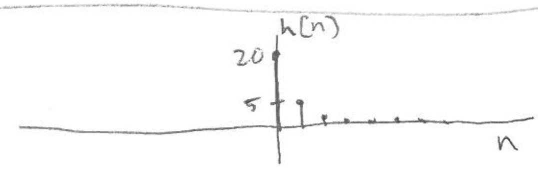


Because $h(t) \neq 0$ for $t < 0$ it is noncausal

Because $\int_{-\infty}^{\infty} |h(t)| dt = 4 < \infty$ it is stable

Because $h(t) \neq 0$ for $t \neq 0$ it has memory

(c)

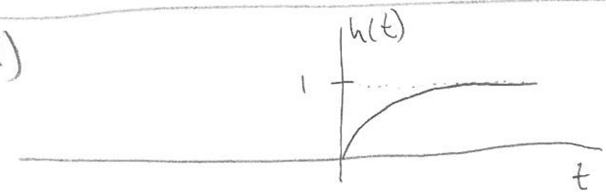


Because $h[n] = 0$ for $n < 0$ it is causal

Because $\sum_{n=0}^{\infty} |h[n]| = \frac{20}{3/4} < \infty$ it is stable

Because $h[n] \neq 0$ for $n \neq 0$ it has memory

(d)



Because $h(t) = 0$ for $t < 0$ it is causal

Because $\int_{-\infty}^{\infty} |h(t)| dt = \infty$ it is unstable

Because $h(t) \neq 0$ for $t \neq 0$ it has memory

2. (30 %) Simplify the following expressions using properties of discrete-time convolution:

(a) $(x[n] + 2\delta[n-1]) * \delta[n-2]$

(b) $(x[n] + 2\delta[n-1]) * (\delta[n+1] + \delta[n-2])$

(c) $(x[n] - u[n-1]) * \delta[n-2]$

(d) $(x[n] - x[n-1]) * u[n]$

(a) $$\left(x[n] + 2\delta[n-1]\right) * \delta[n-2] = x[n] * \delta[n-2] + 2\delta[n-1] * \delta[n-2]$$

$$\boxed{= x[n-2] + 2\delta[n-3]}$$

(b) $$\left(x[n] + 2\delta[n-1]\right) * \left(\delta[n+1] + \delta[n-2]\right) = x[n] * \delta[n+1] + x[n] * \delta[n-2]$$

$$+ 2\delta[n-1] * \delta[n+1] + 2\delta[n-1]\delta[n-2]$$

$$\boxed{= x[n+1] + x[n-2] + 2\delta[n] + 2\delta[n-3]}$$

(c) $$\left(x[n] - u[n-1]\right) * \delta[n-2] = x[n] * \delta[n-2] - u[n-1] * \delta[n-2]$$

$$\boxed{= x[n-2] - u[n-3]}$$

(d) $$\left(x[n] - x[n-1]\right) * u[n] = \sum_{n=-\infty}^{\infty} \left(x[m] - x[m-1]\right) \cdot \underbrace{u[n-m]}$$

this is only "on" when $n-m \geq 0$

$\Rightarrow m \leq n$

$$= \sum_{n=-\infty}^{n} x[m] - x[m-1]$$

$$= x[n] + \sum_{n=-\infty}^{n-1} \cancel{x[m] - x[m]} \rightarrow 0$$

$$\boxed{= x[n]}$$

Alternate solution

$$= x[n] * \underbrace{\left(\delta[n] - \delta[n-1]\right) * u[n]}_{= \delta[n]}$$

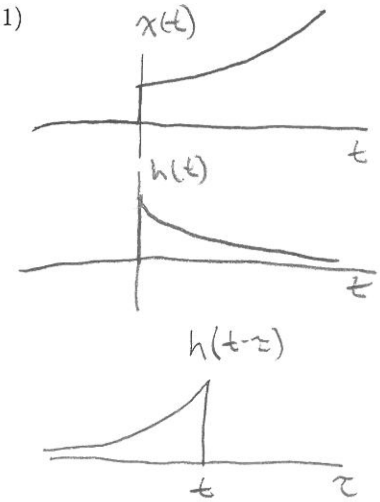
$$= x[n] * \delta[n]$$

$$= x[n]$$

3. (35 %) Determine the output $y(t)$ for the following pairs of signals $x(t)$ and impulse responses $h(t)$:

(a) $x(t) = e^{2t}u(t), \;\; h(t) = e^{-3t}u(t)$

(b) $x(t) = u(t) - 2u(t-1) + u(t-2), \;\; h(t) = u(t+1) - u(t-1)$

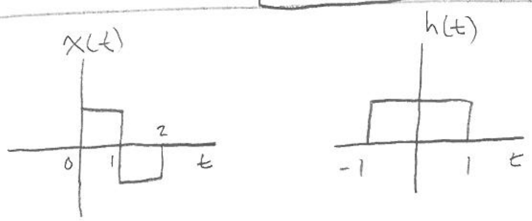


(a) $$x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

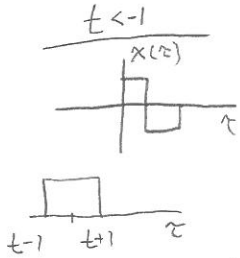
$$= \int_{-\infty}^{\infty} e^{-3\tau} e^{2t} e^{-2\tau}\, \underbrace{u(\tau)}_{\substack{\text{"on"} \\ \text{when} \\ \tau \geq 0}}\, \underbrace{u(t-\tau)}_{\substack{\text{"on"} \\ \text{when} \\ \tau \leq t}}\, d\tau$$

$$= \begin{cases} e^{2t} \displaystyle\int_0^t e^{-5\tau}\, d\tau & t \geq 0 \\ 0 & t < 0 \end{cases}$$

$$\boxed{= \frac{e^{2t}}{5}\left(1 - e^{-5t}\right) u(t)}$$



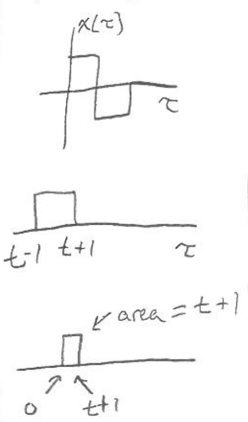
Case 1: $t < -1$



zero overlap

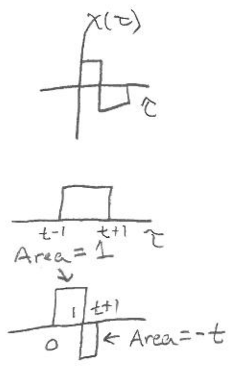
$= 0$

Case 2: $-1 < t < 0$



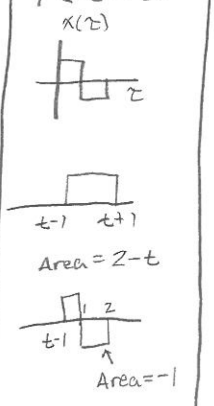
$= t+1$

Case 3: $0 < t < 1$



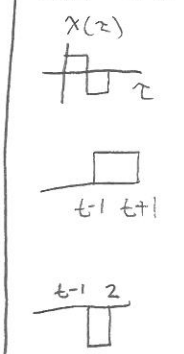
$= 1-t$

Case 4: $1 < t < 2$



$= 1-t$

Case 5: $2 \leq t \leq 3$



$= t-3$

Case 6 $t > 3$

no overlap

$$\boxed{y(t) = \begin{cases} t+1 & -1 \leq t \leq 0 \\ 1-t & 0 < t < 2 \\ t-3 & 2 \leq t \leq 3 \\ 0 & \text{otherwise} \end{cases}}$$