EECS 360 Short Quiz #1

Signal and System Analysis

February 15, 2011

Name: KEY

Open book, open notes, no calculator. Be neat, write legibly. For full credit you must show all work and justify each answer. You may write on the both sides of the paper, and use additional sheets of paper if needed.

1. (30 %) Consider the following signal:

$$x(t) = 3\sin\left(\frac{2\pi(t-T)}{5}\right)$$

Determine the values of T for which the resulting signal is:

(a) an even function of t.

(b) an odd function of t.

(a) cosine is even, and minus cosine is even, so T has to be such that, at $t=0$, the phase of the sin( ) function is $\pm\frac{\pi}{2}, \pm\frac{3\pi}{2} \ldots$

$$\left.\frac{2\pi(t-T)}{5}\right|_{t=0} = \frac{2\pi T}{5} \quad \text{set this equal to } \pm \pi/2$$

$$\frac{2\pi T}{5} = \pm\frac{\pi}{2} \Rightarrow \boxed{T = \pm\frac{5}{4}, \text{ or } \pm 3\cdot\frac{5}{4}, \text{ or } \pm 5\cdot\frac{5}{4} \ldots}$$

(b) same reasoning, except that the phase at $t=0$ needs to be $0, \pm\pi, \pm 2\pi, \pm 4\pi \ldots$

$$\frac{2\pi T}{5} = 0 \Rightarrow T = 0$$

$$= \pm\pi \Rightarrow T = \pm 2\cdot\frac{5}{4}$$

or

$$\pm 4\cdot\frac{5}{4}$$

$$\pm 6\cdot 5/4$$

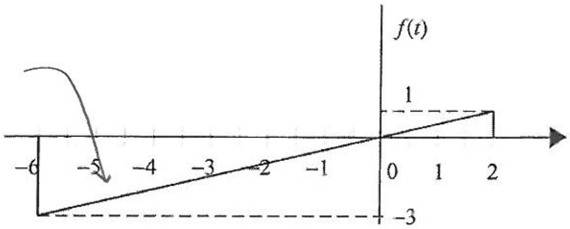
2. (30 %) Consider the signal $f(t)$ shown below.

(a) Write an equation that describes this signal.

(b) Take the derivative of the equation you found in part (a).

(c) Sketch the derivative of this signal.

slope $= \frac{4}{8} = \frac{1}{2}$



(a) we can use a rectangle with a width of 8 that is right-shifted by 2 $\Rightarrow \text{rect}\left(\frac{t+2}{8}\right)$

and we can use a ramp that is right-shifted by 6, amplitude shifted by −3, and has a slope of ½ $\Rightarrow \frac{1}{2}\text{ramp}(t+6) - 3$

$$f(t) = \left(\frac{1}{2}\text{ramp}(t+6) - 3\right)\text{rect}\left(\frac{t+2}{8}\right)$$

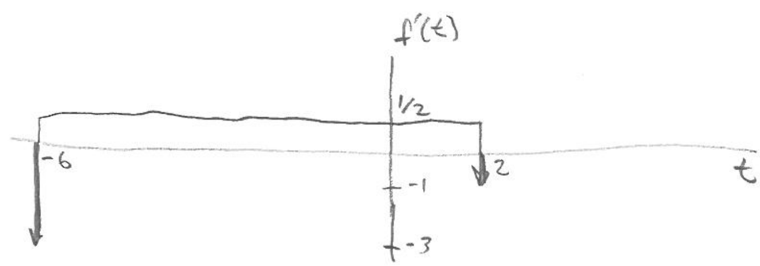
(b) Use the product rule for derivatives $\frac{d}{dt} g(t)h(t) = g'(t)h(t) + g(t)h'(t)$

$$\underbrace{\left(\frac{1}{2}u(t+6)\right)}_{g'(t)}\underbrace{\text{rect}\left(\frac{t+2}{8}\right)}_{h(t)} + \underbrace{\left(\frac{1}{2}\text{ramp}(t-6) - 3\right)}_{g(t)}\underbrace{\left(\delta(t+6) - \delta(t-2)\right)}_{h'(t)}$$

because of these two $\delta(t)$ functions we only need to evaluate this function at the points $t = -6$ and $t = 2$

$$\boxed{f'(t) = \frac{1}{2}u(t+6)\,\text{rect}\left(\frac{t+2}{8}\right) + -3\delta(t+6) - \delta(t-2)}$$

(c)



← we can double-check this answer and see that when we integrate, we get the original $f(t)$ back

3. (40 %) Determine if the following systems are linear, time-invariant, causal, stable, memoryless, and/or invertible.

(a) $y[n] = x[n]u[x[n]]$

(b) $y(t) = \frac{d}{dt}x(t)$

(a) This system is like a diode, the unit step clips all values with a negative amplitude, and passes all values with a positive amplitude

$\boxed{\text{nonlinear}}$ $x[n] \to y[n]$ $\quad -2x[n] \not\to -2y[n]$ because of the clipping

$\boxed{\text{time invariant}}$ $x_1[n] = g[n] \to y_1[n] = x_1[n]\,u[x_1[n]] = g[n]\,u[g[n]]$

$y_1[n-n_0] = \boxed{g[n-n_0]\,u[g[n-n_0]]}$

same

$x_2[n] = g[n-n_0] \to y_2[n] = x_2[n]\,u[x_2[n]] = \boxed{g[n-n_0]\,u[g[n-n_0]]}$

$\boxed{\text{stable}}$ $y[n]$ is bounded between $x[n]$ and zero

$\boxed{\text{causal}}$ and $\boxed{\text{memoryless}}$ To compute $y[n]$, we need only $x[n]$, we do not need any past values, or future values

$\boxed{\text{not invertible}}$ In the process of computing $y[n]$, we destroy all parts of $x[n]$ that have negative amplitude. Once lost, this information cannot be recovered from $y[n]$

(b) $\boxed{\text{linear}}$ let $x(t) = \alpha g(t) + \beta h(t) \to y(t) = \alpha \frac{d}{dt}g(t) + \beta \frac{d}{dt}h(t)$

$\boxed{\text{time invariant}}$ $x_1(t) = g(t) \to y_1(t) = \frac{d}{dt}x_1(t) = \frac{d}{dt}g(t)$

$y_1(t-t_0) = \boxed{\frac{d}{dt}g(t-t_0)}$ same

$x_2(t) = g(t-t_0) \to y_2(t) = \frac{d}{dt}x_2(t) = \boxed{\frac{d}{dt}g(t-t_0)}$

? $\boxed{\text{causal}}$ and $\boxed{\text{memoryless}}$ recall the definition of the derivative from your first day of calculus $y(t) = \lim_{\Delta t \to 0} \frac{x(t) - x(t-\Delta t)}{\Delta t}$ In the limit there is no memory

? $\boxed{\text{not invertible}}$ constants map to same value of zero

? $\boxed{\text{invertible}}$ to get $x(t)$ back, simply integrate $\Rightarrow x(t) = \int_{-\infty}^{t} y(\tau)\,d\tau$

? $\boxed{\text{stable}}$ $\to$ more easily proved with Laplace transform