EECS 360 Short Quiz #1

Signal and System Analysis

September 13, 2016

Name: KEY

Open book, open notes, no calculator. Be neat, write legibly. For full credit you must show all work and justify each answer. You may write on the both sides of the paper, and use additional sheets of paper if needed.

1. (20 %) In each of the following cases, simplify the expression as much as possible using the properties of the continuous-time unit impulse. Provide some explanation or intermediate steps for each answer.

   (a) $e^{-(t-4)}u(t-4)\delta(t-5)$

   (b) $\int_{-\infty}^{t-5}\delta(\tau-1)\,d\tau$

   (c) $\frac{d}{dt}[e^{-(t-4)}u(t-4)]$

   (d) $\int_{-\infty}^{\infty}x(\tau)\delta(t-\tau)\,d\tau$

(a) $e^{-(t-4)}u(t-4)\underbrace{\delta(t-5)}_{\text{sifting property means evaluate everything @ } t=5} = \cancel{e^{-(5-4)}}^{\,e^{-1}}\,\cancel{u(5-4)}^{\,1}\,\delta(t-5)\ \boxed{= e^{-1}\delta(t-5)}$

(b) $\delta(t) \underset{d/dt}{\overset{\int}{\rightleftarrows}} u(t) \Rightarrow \int_{-\infty}^{t-5}\delta(\tau-1)\,d\tau = u(t-5-1)\ \boxed{= u(t-6)}$

(c) recall the product rule for derivatives

$$\frac{d}{dt}\left[e^{-(t-4)}u(t-4)\right] = \left[\frac{d}{dt}e^{-(t-4)}\right]u(t-4) + e^{-(t-4)}\left[\frac{d}{dt}u(t-4)\right]$$

$$= -e^{-(t-4)}u(t-4) + \underbrace{e^{-(t-4)}}_{\text{evaluate @ } t=-4}\delta(t-4)$$

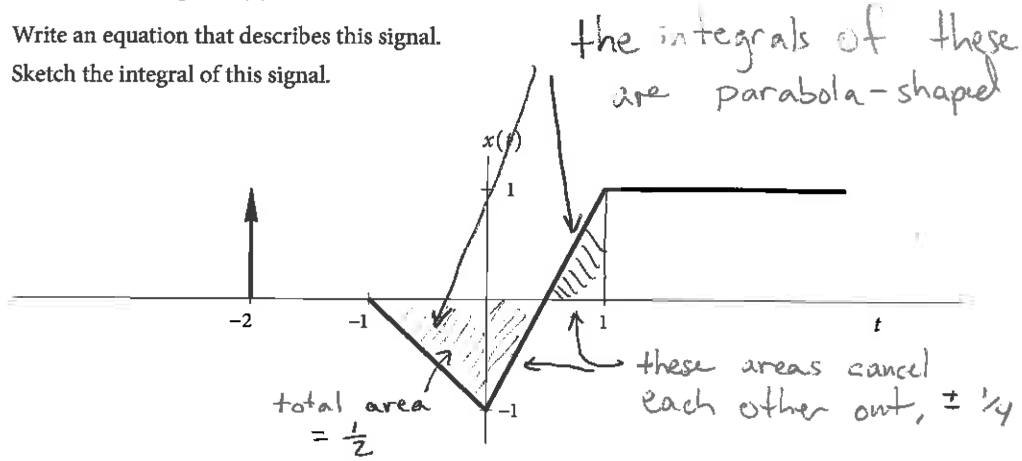
$$\boxed{= -e^{-t-4}u(t-4) + \delta(t-4)}$$

(d) $\int_{-\infty}^{\infty}x(\tau)\underbrace{\delta(t-\tau)}_{t-\tau=0 \Rightarrow \tau=t}\,d\tau = \boxed{x(t)}$

2. (30 %) Consider the signal $x(t)$ shown below.

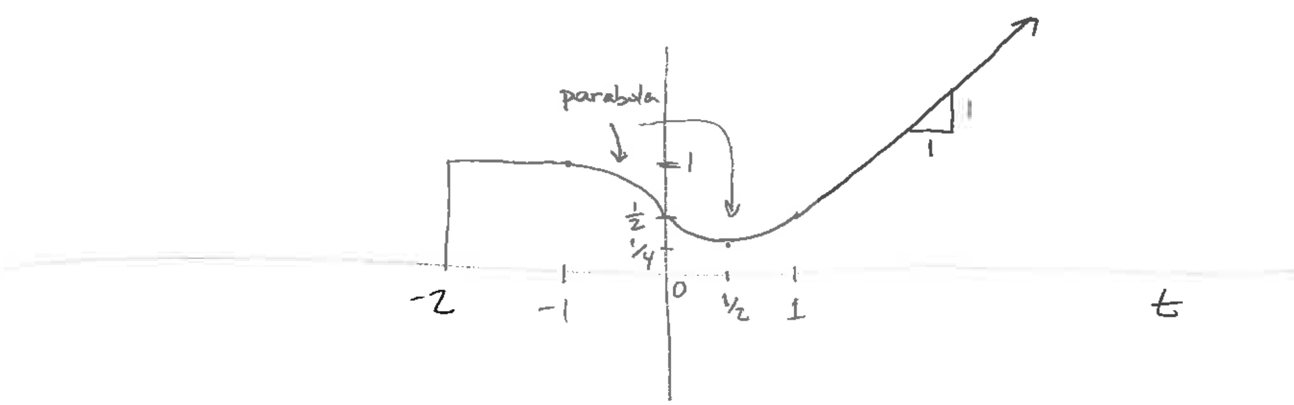
(a) Write an equation that describes this signal.

(b) Sketch the integral of this signal.



(a) $\delta(t+2) - \mathrm{ramp}(t+1) + 3\,\mathrm{ramp}(t) - 2\,\mathrm{ramp}(t-1)$

(b)



50%

3. ~~(40 %)~~ System classification: state whether or not the following systems are (i) linear, (ii) time-invariant, (iii) stable, and (iv) causal. Provide convincing justification of each claim. To show that a property does not hold, all you must do is give an example input whose output does not satisfy the condition of the property.

(a) A phase modulator: $y(t) = \cos[\omega_c t + x(t)]$.

(b) An amplitude modulator: $y(t) = [A + x(t)]\cos(\omega_c t)$.

(a) (i) to be linear, if the input is zero the output should be zero

$y(t) = \cos(\omega_c t + 0) = \cos(\omega_c(t)) \neq 0 \Rightarrow$ $\boxed{\text{non-linear}}$

(ii) time invariant? the system needs to know t to operate $\cos(\omega_c t)$

and so $x_1(t) = g(t) \rightarrow y_1(t) \xrightarrow[\text{delay}]{} y_1(t-t_0)$ $\boxed{\text{time varying}}$

gives a different result than $x_2(t) = g(t-t_0)$

(iii) stable? the cos[ ] function is always between ± 1

(for real arguments) therefore the system output is always

bounded $\Rightarrow$ $\boxed{\text{stable}}$

(iv) causal? the system is actually memoryless [it only needs $x(t)$

and the current time] so it is $\boxed{\text{causal}}$

(b) (i) likewise $x(t) = 0 \Rightarrow y(t) = A\cos(\omega_c t) \neq 0$ $\boxed{\text{nonlinear}}$

(ii) $\boxed{\text{time varying}}$ for the same reason as above

(iii) if $|x(t)| < B$ then $|A + x(t)| < B + A$

and we already know $|\cos(\cdot)| < 1 \Rightarrow |y(t)| < B + A$

$\boxed{\text{stable}}$

(iv) causal? once again, it is memoryless, so it is $\boxed{\text{causal}}$