EECS 360 Short Quiz #3
Signal and System Analysis
November 4, 2014

Name: KEY

Open book, open notes, no calculator. Be neat, write legibly. For full credit you must show all work and justify each answer. You may write on the both sides of the paper, and use additional sheets of paper if needed.

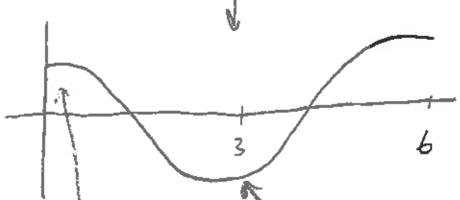
1. (40 %) You are given the following information about a signal $x(t)$:
   (a) $x(t)$ is a real signal (i.e., it has no imaginary part). ← all terms can be written as $A\cos(Bt+c)$
   (b) $x(t)$ is periodic with period $T = 6$ and has Fourier series coefficients $c_x[k]$.
   (c) $c_x[k] = 0$ for $k = 0$ and $k > 2$. ← there might be non-zero values for $C_x[-2], C_x[-1], C_x[1], C_x[2]$
   (d) $x(t) = -x(t-3)$.
   (e) $\frac{1}{6}\int_{-3}^{3} |x(t)|^2\, dt = \frac{1}{2}$. by Parseval's theorem we have $|C_x[-2]|^2 + |C_x[-1]|^2 + |C_x[1]|^2 + |C_x[2]|^2 = \frac{1}{2}$
   (f) $c_x[1]$ is a positive real number.

   Show that $x(t) = A\cos(Bt + C)$, and determine the values of the constants A, B, and C.

We are left with

$$\underbrace{A_1\cos(2\pi t/6 + C_1)}_{\substack{\text{This has the } k=\pm 1 \\ \text{terms of } C_x[k]}} + \underbrace{A_2\cos(2\pi\cdot 2\cdot t/6 + C_2)}_{\substack{\text{this has the} \\ k=\pm 2 \text{ terms} \\ \text{of } C_x[k]}}$$

↓ this will look something like ↓



notice that $x(t)$ is equal to $-x(t-3)$

this matches clue (d)

↓ this will look something like ↓

notice that $x(t)$ is equal to $+x(t-3)$

this violates clue (d) ⇒ therefore $C_x[-2]$ and $C_x[2]$ are zero!

Clue (f) says that $C_x[1]$ and $C_x[-1]$ have no phase

⇒ therefore $C_1 = 0$

⇒ We now know the answer ⇒ $\boxed{x(t) = \cos(2\pi t/6) = \frac{1}{2}e^{j2\pi/6} + \frac{1}{2}e^{-j2\pi/6}}$

$\left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 = \frac{1}{2}$ ← parseval's

2. (20 %) Let $x(t)$ be any signal with Fourier transform $X(j\omega)$ [or $X(f)$ if you prefer]. The frequency-shift property of the Fourier transform may be stated as

$$e^{j\omega_0 t}x(t) \overset{\mathcal{F}}{\longleftrightarrow} X(j(\omega-\omega_0)).$$

(a) Prove the frequency-shift property by applying the frequency shift to the analysis equation

$$X(j\omega) = \int_{-\infty}^{+\infty} x(t)e^{-j\omega t}\,dt.$$

(b) Prove the frequency-shift property by utilizing the Fourier tranform of $e^{j\omega_0 t}$ in conjunction with the multiplication property of the Fourier transform.

(a)

$$X(j(\omega-\omega_0)) = \int_{-\infty}^{\infty} x(t)\,e^{-j(\omega-\omega_0)t}\,dt = \int_{-\infty}^{\infty} e^{j\omega_0 t}x(t)e^{-j\omega t}\,dt \boxed{= \mathcal{F}\{e^{j\omega_0 t}x(t)\}}$$

therefore

$$e^{j\omega_0 t} \overset{\mathcal{F}}{\longleftrightarrow} X(j(\omega-\omega_0))$$

(b)

$$e^{j\omega_0 t}\cdot x(t) \overset{\mathcal{F}}{\longleftrightarrow} \frac{1}{2\pi}\mathcal{F}\{e^{j\omega_0 t}\} * X(j\omega)$$

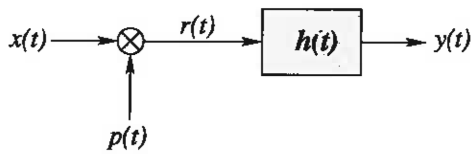
$$= \frac{1}{2\pi}\left[2\pi\delta(\omega-\omega_0)\right] * X(j\omega)$$

using the property $x(t) * A\delta(t-t_0) = A\,x(t-t_0)$

we have

$$e^{j\omega_0 t}\cdot x(t) \longleftrightarrow X(j(\omega-\omega_0))$$

3. (40 %) Consider the system depicted below



$4\,\text{sinc}(4t)$
↓

where $x(t) = \frac{\sin(4\pi t)}{\pi t}$, $p(t) = 2\cos(2\pi t)$, and the impulse response $h(t)$ is given by $h(t) = 1+3\sin(4\pi t)+2\cos(8\pi t)$.

(a) Provide a labeled sketch of $R(f)$ [or $R(j\omega)$ if you prefer], the Fourier transform of $r(t)$.

(b) Provide a labeled sketch of $Y(f)$ [or $Y(j\omega)$ if you prefer], the Fourier transform of $y(t)$.

(c) Determine $y(t)$.

$X(f) = \text{rect}(f/4)$

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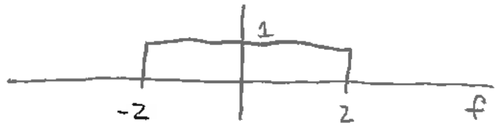
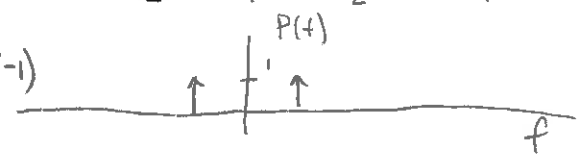
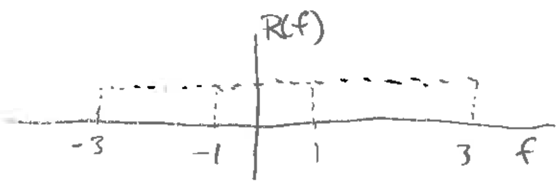


$P(f) = \delta(f+1) + \delta(f-1)$

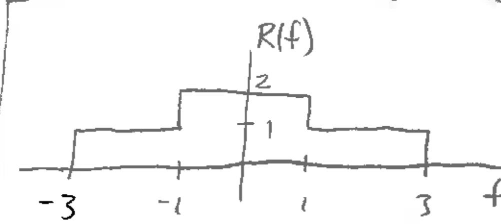


(a) Because $x(t)$ and $p(t)$ are multiplied in the time domain, $X(f)$ and $P(f)$ are convolved in the freq domain

$\Rightarrow$ convolve the two sketches above to get $R(f)$

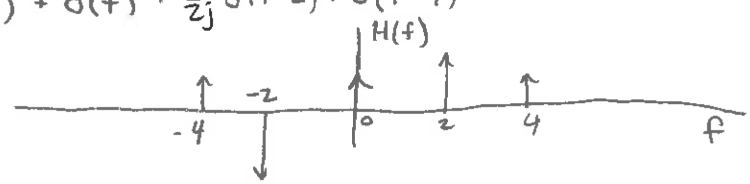


$\Rightarrow$

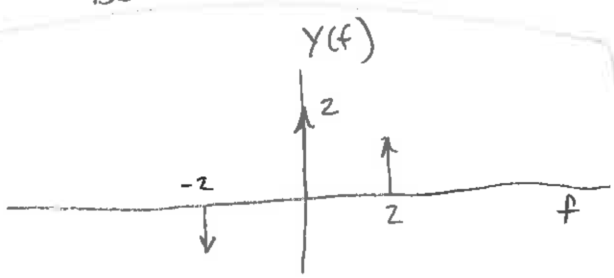


(b)

$H(f) = \delta(f+4) - \frac{3}{2j}\delta(f+2) + \delta(f) + \frac{3}{2j}\delta(f-2) + \delta(f-4)$



Because $r(t)$ and $h(t)$ are convolved in the time domain $\Rightarrow$ Multiply sketch of $R(f)$ and $H(f)$ to get $Y(f)$



(c)

$$\boxed{y(t) = 2 + 3\sin(4\pi t)}$$