EECS 360 Short Quiz #2

Signal and System Analysis

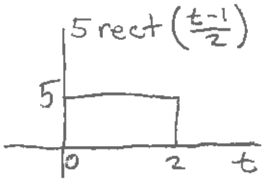
October 7, 2014

Name: KEY

Open book, open notes, no calculator. Be neat, write legibly. For full credit you must show all work and justify each answer. You may write on the both sides of the paper, and use additional sheets of paper if needed.

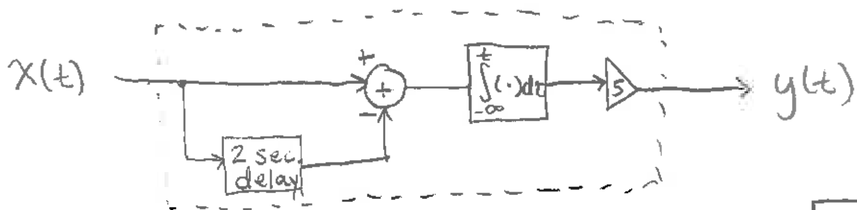
1. (45 %) An engineer wants to build a system that has the impulse response $h(t) = 5\mathrm{rect}\left(\frac{t-1}{2}\right)$.
   (a) Plot $h(t)$. Carefully label each axis.
   (b) Show how to construct the system using a delay, an integrator, gains, and adders.
   (c) Find an expression for the response of this system $y(t)$ when the excitation is an arbitrary waveform $x(t)$.
   (d) Is this system causal? Justify your answer.
   (e) Is this system stable? Justify your answer.

(a)



$$h(t) = \begin{cases} 0 & t<0 \\ 5/2 & t=0 \\ 5 & 0<t<2 \\ 5/2 & t=2 \\ 0 & t>2 \end{cases} = \underbrace{5\big(u(t)-u(t-2)\big)}_{\text{this is another way to write it}}$$

(b)



the main idea we start with is that $\rightarrow \boxed{\int_{-\infty}^{t}(\cdot)d\tau} \rightarrow$ has an impulse response of $u(t)$. The gain of 5 and the delayed $u(t-2)$ follow from there.

(c)
$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)\,d\tau = 5\int_{-\infty}^{\infty} x(\tau)\underbrace{u(t-\tau)}_{\substack{\text{this is 1}\\ \text{for } -\infty<\tau<t}}d\tau - 5\int_{-\infty}^{\infty} x(\tau)\underbrace{u(t-\tau-2)}_{\substack{\text{this is 1}\\ \text{for } -\infty<\tau<t-2}}d\tau$$

$$= 5\int_{-\infty}^{t} x(\tau)d\tau - 5\int_{-\infty}^{t-2} x(\tau)d\tau \boxed{= 5\int_{t-2}^{t} x(\tau)d\tau}$$

(d) Because the plot in (a) is zero for $t<0$, the system is causal

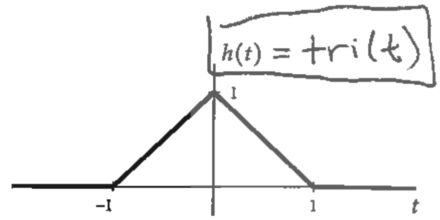
(e) Because $\int_{-\infty}^{\infty} |h(t)|\,dt = 10$, which is less than ∞, it is stable

2. (35 %) An LTI system has the impulse response $h(t)$ shown below. Use linearity and time invariance to determine the system output $y(t)$ if the input $x(t)$ is:

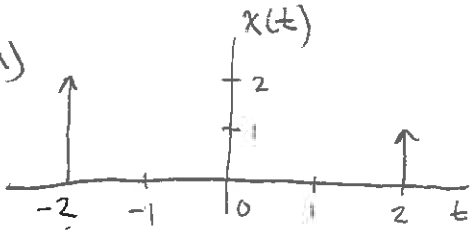
(a) $x(t) = 2\delta(t+2) + \delta(t-2)$

(b) $x(t) = \delta(t-1) + \delta(t-2) + \delta(t-3)$

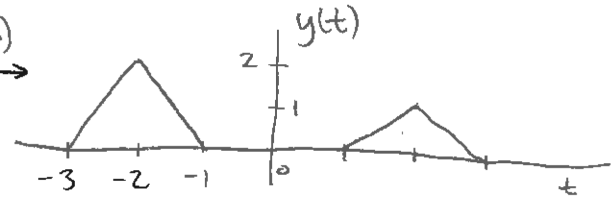
(c) $x(t) = \sum_{n=0}^{\infty}(-1)^n \delta(t-2n)$



(a)

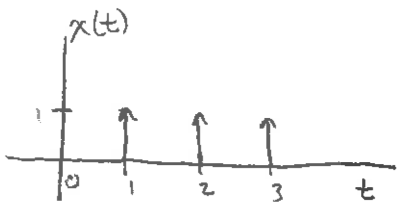


$\xrightarrow{h(t)}$

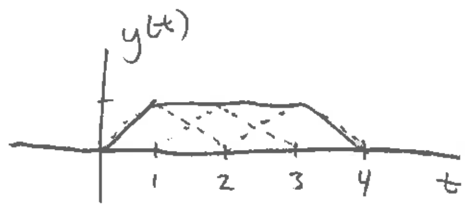


$$y(t) = 2h(t+2) + h(t-2)$$
$$= 2\,\mathrm{tri}(t+2) + \mathrm{tri}(t-2)$$

(b)

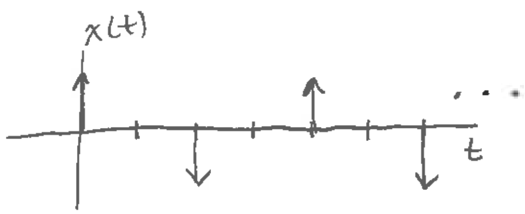


$\xrightarrow{h(t)}$

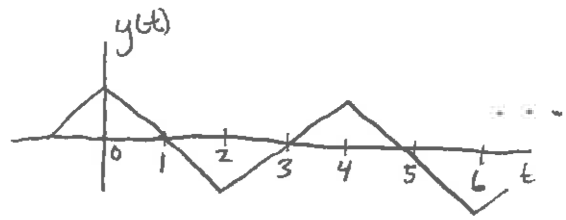


$$y(t) = h(t-1) + h(t-2) + h(t-3)$$
$$= \mathrm{tri}(t-1) + \mathrm{tri}(t-2) + \mathrm{tri}(t-3)$$

(c)



$\xrightarrow{h(t)}$



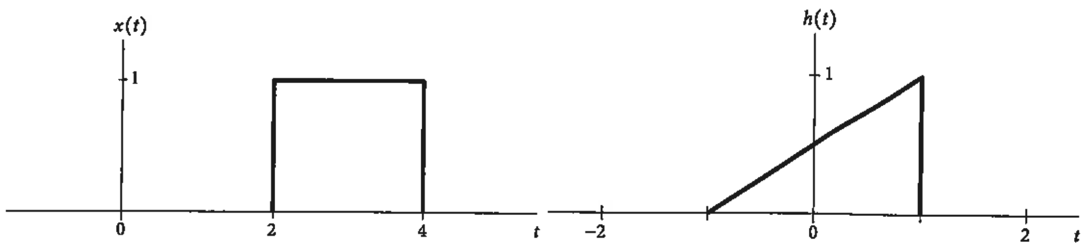
$$y(t) = \sum_{n=0}^{\infty}(-1)^n h(t-2n)$$
$$= \sum_{n=0}^{\infty}(-1)^n \mathrm{tri}(t-2n)$$

3. (20 %) Without performing the convolutions, find the left- and right-edge locations and the widths of the signals $y(t)$ that result from the convolutions of:

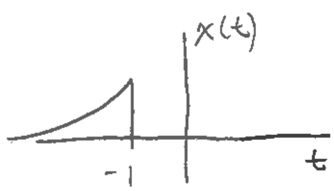
(a)

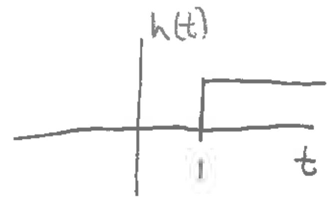
$$x(t) = \begin{cases} 0, & t > -1 \\ e^{t}, & t \le -1 \end{cases} \qquad \text{and} \qquad h(t) = \begin{cases} 0, & t < 1 \\ 1, & t \ge 1 \end{cases}$$

(b) $x(t)$ and $h(t)$ as shown in the plots below.



(a)





$t_{x_0} = -\infty$

$t_{x_1} = -1$

$\Delta t_x = \infty$

$t_{h_0} = 1$

$t_{h_1} = +\infty$

$\Delta t_h = \infty$

$$t_{y_0} = t_{x_0} + t_{h_0} = -\infty + 1 = -\infty$$

$$t_{y_1} = t_{x_1} + t_{h_1} = -1 + \infty = +\infty$$

$$\Delta t_y = \Delta t_x + \Delta t_h \boxed{= \infty}$$

(b) Inspecting the given plots yields

$t_{x_0} = 2$ $\quad t_{h_0} = -1$

$t_{x_1} = 4$ $\quad t_{h_1} = +1$

$\Delta t_x = 2$ $\quad \Delta t_h = 2$

$\Rightarrow$

$$t_{y_0} = t_{x_0} + t_{h_0} = 2 - 1 = 1$$

$$t_{y_1} = t_{x_1} + t_{h_1} = 4 + 1 = 5$$

$$\Delta t_y = \Delta t_x + \Delta t_h = 2 + 2 = 4$$