EECS 360 Short Quiz #1

Signal and System Analysis

September 6, 2012

Name: KEY

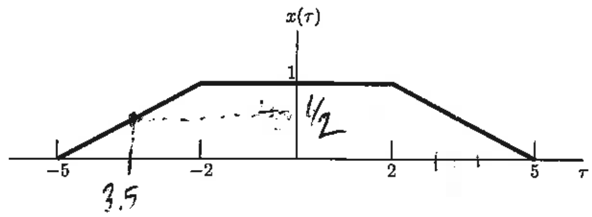
Open book, open notes, no calculator. Be neat, write legibly. For full credit you must show all work and justify each answer. You may write on the both sides of the paper, and use additional sheets of paper if needed.

1. (30 %) Evaluate the following expressions:

(a) $\int_{-\infty}^{\infty} \delta(\tau - 2)\tau^3 \, d\tau$.

(b) $\int_{-\infty}^{\infty} \delta(\tau - a)\delta(t + \tau - b) \, d\tau$.

(c) $\int_{-\infty}^{\infty} \delta(2\tau + 7)x(\tau) \, d\tau$, where $x(\tau)$ is shown to the right.



(d) $\sum_{k=-\infty}^{\infty} \delta[2k - 8] \cos\left(\frac{\pi}{16}k\right)$.

(a) $\int_{-\infty}^{\infty} \delta(\underbrace{\tau-2}_{})\tau^3 \, d\tau = \tau^3\Big|_{\tau=2} \boxed{= 8}$

$\tau - 2 = 0 \Rightarrow \tau = 2$

use the identity $\delta(a(t-t_0)) = \frac{1}{|a|}\delta(t-t_0)$

(b) $\int_{-\infty}^{\infty} \delta(\tau-a)\delta(\underbrace{t+\tau-b}_{})d\tau = \delta(\tau-a)\Big|_{\tau=-t+b}$ $\boxed{\begin{aligned} &= \delta(-t+b-a) \\ &= \delta(t-b+a) \end{aligned}}$

$t + \tau - b = 0 \Rightarrow \tau = -t + b$

(c) $\int_{-\infty}^{\infty} \delta(2\tau+7)x(\tau)d\tau = \int_{-\infty}^{\infty} \frac{1}{2}\delta(\underbrace{\tau+3.5}_{})x(\tau)d\tau = \frac{1}{2}x(\tau)\Big|_{\tau=-3.5} \boxed{= \frac{1}{4}}$

$\tau + 3.5 = 0 \rightarrow \tau = -3.5$

(d) $\sum_{K=-\infty}^{\infty} \delta[\underbrace{2K-8}_{}]\cos\left(\frac{\pi}{16}K\right) = \cos\left(\frac{\pi}{16}K\right)\Big|_{K=4} \boxed{= \cos\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}}$

$2K - 8 = 0$

$\Rightarrow K = 4$

2. (40 %) The *moving average* $y(t)$ of a signal $x(t)$ is given by the equation

$$y(t) = \frac{1}{T}\int_t^{t+T} x(\tau)\, d\tau$$

where T is the time over which the average is made.

(a) Is the moving average a linear system? Justify your answer.

(b) Is the moving average a time-invariant system? Justify your answer.

(c) Is the moving average a causal system? Justify your answer.

(d) Is the moving average a BIBO stable system? Justify your answer.

(a) Conduct two reference experiments $x_1(t) \longrightarrow y_1(t) = \frac{1}{T}\int_t^{t+T} x_1(\tau)d\tau$

$$x_2(t) \longrightarrow y_2(t) = \frac{1}{T}\int_t^{t+T} x_2(\tau)d\tau$$

Now superimpose the two

$$x_3(t) = \alpha x_1(t) + \beta x_2(t) \longrightarrow y_3(t) = \frac{1}{T}\int_t^{t+T}(\alpha x_1(\tau) + \beta x_2(\tau))d\tau$$

$$= \alpha \frac{1}{T}\int_t^{t+T} x_1(\tau)d\tau + \beta\frac{1}{T}\int_t^{t+T} x_2(\tau)d\tau$$

$$\boxed{= \alpha y_1(t) + \beta y_2(t) \Leftarrow \text{Linear}}$$

(b) Let $x_1(t) = g(t) \longrightarrow y_1(t) = \frac{1}{T}\int_t^{t+T} x_1(\tau)d\tau = \frac{1}{T}\int_t^{t+T} g(\tau)d\tau \xrightarrow[t_0]{\text{delay}} \boxed{y_1(t-t_0) = \frac{1}{T}\int_{t-t_0}^{t-t_0+T} x(\tau)d\tau}$

Let $x_2(t) = g(t-t_0) \longrightarrow y_2(t) = \frac{1}{T}\int_t^{t+T} x_2(\tau)d\tau$

$$= \frac{1}{T}\int_t^{t+T} g(\tau - t_0)d\tau \qquad \boxed{= \frac{1}{T}\int_{t-t_0}^{t-t_0+T} x(\lambda)d\lambda}$$

Same

Change of variables Let $\lambda = \tau - t_0$

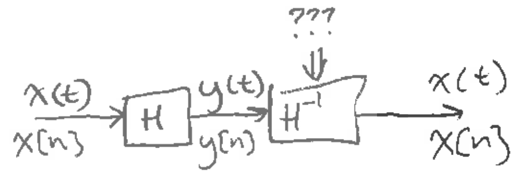
$\Rightarrow \tau = \lambda + t_0$

$\Rightarrow d\lambda = d\tau$

$\boxed{\text{Time invariant}}$

(c) To compute $y(t)$ we must have $x(t)$ through $x(t+T)$ at time t, $x(t+T)$ is in the future $\boxed{\text{non-causal}}$

(d) Unlike the running integral, $\int_{-\infty}^{t} x(\tau)d\tau$, the moving average has the ability to forget. Just like any other average, the output value lies somewhere inbetween the input, so a bounded input always has a bounded output



3. (30 %) Determine if the following systems are invertible. If yes, find the inverse systems.

   (a) $y[n] = 3x[2n+5]$

   (b) $y(t) = \int_{\infty}^{t} x(\tau + 4)\, d\tau$ $-\infty$

   (c) $4\dfrac{dy(t)}{dt} + 2y(t) = x(t)$

   (d) $y[n] = \cos(2\pi x[n])$

(a) The output sequence, $y[n]$, consists of the odd samples of $x(n)$. The even-indexed samples are not found in $y[n]$ so $x[n]$ cannot be recovered from $y[n]$, $\boxed{\text{not invertible}}$

(b) $y(t)$ consists of a time advance of 4 seconds, and a running integral. To get $x(t)$ back, simply differentiate $y(t)$ and insert a time delay of 4 seconds

$$\boxed{x(t) = \frac{d}{dt} y(t-4)}$$

(c) As it is written, the differential equation tells you exactly how to obtain $x(t)$ from $y(t)$

$$\boxed{x(t) = 4\frac{d}{dt}y(t) + 2y(t)}$$

(d) The cosine function has a many-to-one mapping, for example $\cos(0) = \cos(2\pi) = 1$

Therefore $x[n]$ cannot be obtained from $y[n]$

$\boxed{\text{not invertible}}$