A causal LTI system is described by the difference equation

$$y[n] = y[n-1] + y[n-2] + x[n-1]$$

(a) Find the system function $H(z) = Y(z)/X(z)$ for this system.

(b) Plot the poles and zeros of $H(z)$ and indicate the ROC

(c) Find the impulse response of the system

(d) You should have found the system to be unstable, Find a stable (non-causal) impulse response that satisfies the difference equation

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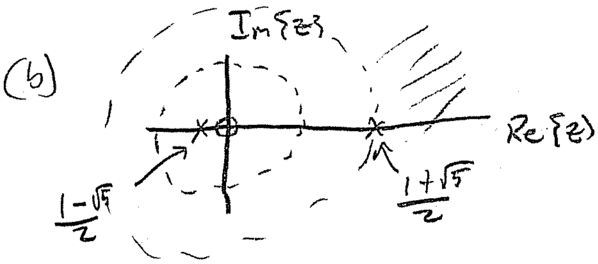
(a) $$H(z) = \frac{Y(z)}{X(z)} = \frac{z^{-1}}{1 - z^{-1} - z^{-2}}$$

Poles are at $\underbrace{\frac{1}{2} \pm \frac{\sqrt{5}}{2}}_{\text{these are both \underline{real}!}}$

(b)



ROC: $|z| > \frac{1}{2} + \frac{\sqrt{5}}{2}$

(c) $$H(z) = \frac{1/\sqrt{5}}{1 - \left(\frac{1+\sqrt{5}}{2}\right)z^{-1}} - \frac{1/\sqrt{5}}{1 - \left(\frac{1-\sqrt{5}}{2}\right)z^{-1}}$$

$$h[n] = \underbrace{\frac{1}{\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right)^n u[n]}_{\text{this one blows up as } n \to \infty} - \frac{1}{\sqrt{5}}\left(\frac{1-\sqrt{5}}{2}\right)^n u[n]$$

(d) To be stable, the ROC must be $\left|\frac{1-\sqrt{5}}{2}\right| < |z| < \left|\frac{1+\sqrt{5}}{2}\right|$

and we get

$$h[n] = \underbrace{\frac{1}{5}\left(\frac{1+\sqrt{5}}{2}\right)^n u[-n-1]}_{\text{Left-sided}} + \underbrace{\frac{1}{5}\left(\frac{1-\sqrt{5}}{2}\right)^n u[n]}_{\text{right-sided}}$$

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