A causal LTI system is described by the difference equation

\[ y[n] = y[n-1] + y[n-2] + x[n-1] \]

(a) Find the system function \( H(z) = \frac{Y(z)}{X(z)} \) for this system.
(b) Plot the poles and zeros of \( H(z) \) and indicate the ROC.
(c) Find the impulse response of the system.
(d) You should have found the system to be unstable. Find a stable (non-causal) impulse response that satisfies the difference equation.

(a) 
\[
H(z) = \frac{Y(z)}{X(z)} = \frac{z^{-1}}{1 - \frac{1}{z^2} - \frac{1}{z^4}} \quad \text{Poles are at } \frac{1}{z} \pm \frac{\sqrt{5}}{2} \text{!}
\]

(b) 
ROC: \( |z| > \frac{1}{2} + \frac{\sqrt{5}}{2} \)

(c) 
\[
H(z) = \frac{-\sqrt{5}}{1 - (1 + \sqrt{5})z^{-1}} - \frac{\sqrt{5}}{1 - (1 - \sqrt{5})z^{-1}}
\]

\[
h[n] = \frac{1}{\sqrt{5}} (1 + \sqrt{5})u[n] - \frac{1}{\sqrt{5}} (1 - \sqrt{5})u[n]
\]

- this one blows up as \( n \to \infty \)

(d) To be stable, the ROC must be \( \left| \frac{1 - \sqrt{5}}{2} \right| < |z| < \left| \frac{1 + \sqrt{5}}{2} \right| \)

and we get

\[
h[n] = \frac{1}{5} (1 + \sqrt{5})u[n-1] + \frac{1}{5} (1 - \sqrt{5})u[n] \quad \text{(left-sided)}
\]

\[
h[n] = \frac{1}{5} (1 + \sqrt{5})u[-n-1] + \frac{1}{5} (1 - \sqrt{5})u[-n] \quad \text{(right-sided)}
\]