Assume that \(x[n]\) is real and even, i.e. \(x[n] = x[-n]\).
Further assume that \(z_0\) is a zero of \(X(z)\), i.e. \(X(z_0) = 0\).

(a) Show that \(1/z_0\) is also a zero of \(X(z)\).
(b) Are there other zeros of \(X(z)\) implied by the information given?

(a) According to the definition of the z-transform, we have
\[
\sum_{n=-\infty}^{\infty} x[n] z^{-n} = X(z)
\]
and we also have
\[
\sum_{n=-\infty}^{\infty} x[-n] z^{-n} = \sum_{m=-\infty}^{\infty} x[n] z^{-n} = X(z^{-1}) = X(z)
\]
Let \(n = -m\)

If we expand \(X(z)\) as a factored rational polynomial we have
\[
X(z) = X(z^{-1}) = \frac{(z-z_0)(z-z_1)\cdots(z-z_m)}{(z-p_0)(z-p_1)\cdots(z-p_N)} = \frac{(z^{-1}-z_0)(z^{-1}-z_1)\cdots(z^{-1}-z_m)}{(z^{-1}-p_0)(z^{-1}-p_1)\cdots(z^{-1}-p_N)}
\]

The only way this can be true is if the zeros (and poles!) of \(X(z)\) come in pairs \(z = z_0\) and \(z = 1/z_0\).

(b) Yes, since \(x[n]\) is given to be real, its poles and zeros come in complex-conjugate pairs, so if you are given a zero \(z = z_0\), you also know there are zeros at \(z = z_0^*\), \(z = 1/z_0\) and \(z = 1/z_0^*\).