Consider a continuous time LTI system for which the excitation $x(t)$ and response $y(t)$ are related by the differential equation

$$\frac{d^2}{dt^2} y(t) - \frac{d}{dt} y(t) - 2y(t) = x(t) \qquad (1)$$

Let $X(s)$ and $Y(s)$ denote Laplace transforms of $x(t)$ and $y(t)$, respectively, and let $H(s)$ denote the Laplace transform of $h(t)$, the impulse response of the system

(a) Determine $H(s)$. Sketch the pole-zero plot of $H(s)$

(b) Determine $h(t)$ for each of the three following cases:

1) The system is stable

2) The system is causal

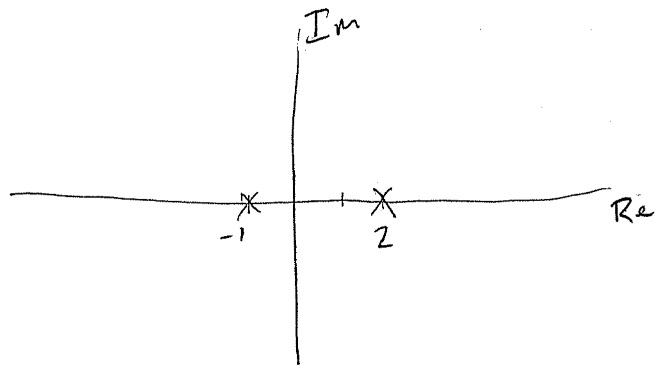
3) the system is <u>neither</u> causal <u>nor</u> stable

(a) Take the Laplace transform of (1), solve for $H(s) = \frac{Y(s)}{X(s)}$

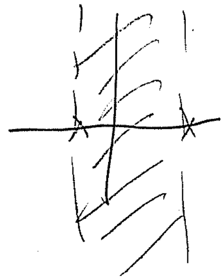
$$s^2 Y(s) - s\,Y(s) - 2\,Y(s) = X(s)$$

$$Y(s)\left[s^2 - s - 2\right] = X(s)$$

$$\Rightarrow \frac{Y(s)}{X(s)} \triangleq H(s) = \frac{1}{s^2 - s - 2} = \frac{1}{(s-2)(s+1)}$$



(b) (1) The system is stable. This means that the ROC includes the $j\omega$ axis



$$H(s) = \frac{A}{s-2} + \frac{B}{s+1}$$

$$A = \frac{1}{2+1} = \frac{1}{3}$$

$$B = \frac{1}{-1-2} = -\frac{1}{3}$$

$$H(s) = \frac{1}{3}\frac{1}{s-2} - \frac{1}{3}\frac{1}{s+1}$$

← both of these are in the O+W tables

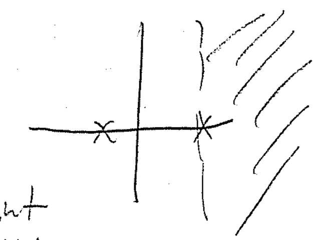
← this corresponds to a right-sided function of time

this is a left-sided function of time

$$\boxed{h(t) = -\frac{1}{3}e^{2t}u(-t) - \frac{1}{3}e^{-t}u(t)}$$

← this is definitely stable (it integrates to a finite number) but it is non-causal

(2) The system is causal. This means that the ROC is to the right of the right-most pole



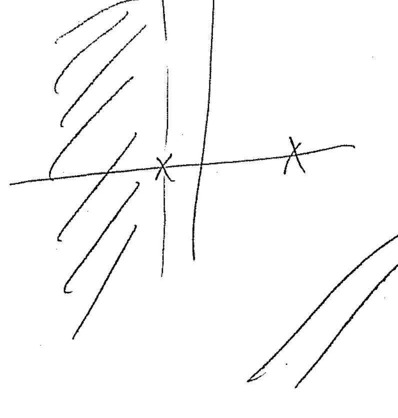
$$H(s) = \frac{1}{3}\frac{1}{s-2} - \frac{1}{3}\frac{1}{s+1}$$

both correspond to right sided functions of time

$$\boxed{h(t) = \frac{1}{3}e^{2t}u(t) - \frac{1}{3}e^{-t}u(t)}$$

← definitely causal, but first term "blows up" so it's unstable

(3) Neither causal nor stable. This means the ROC does not include $j\omega$ axis, and ROC is not to the right of right-most pole



$$\boxed{h(t) = -\frac{1}{3}e^{2t}u(-t) + \frac{1}{3}e^{-t}u(-t)}$$

↑

definitely not stable, and not causal either

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