A pressure gauge that can be modeled as an LTI system has a time response to a unit step input given by \((1 - e^{-t} - te^{-t})u(t)\). For a certain input \(x(t)\), the output is observed to be \((2 - 3e^{-t} + e^{-3t})u(t)\).

For this observed measurement, determine the true pressure input to the gauge as a function of time.

**Experiment 1 was conducted**

\[
\begin{align*}
\text{U(t)} & \rightarrow \text{Gauge} & S(t) &= (1 - e^{-t} - te^{-t})u(t) \\
\end{align*}
\]

We know that the step response \(S(t)\) is the integral of the impulse response \(h(t)\), so...

\[
\begin{align*}
h(t) &= \frac{d}{dt} S(t) = (0 + e^{-t} + te^{-t} - e^{-t})u(t) + \left(1 - e^{-t} - te^{-t}\right)\delta(t) \\
&= te^{-t}u(t)
\end{align*}
\]

**Experiment 2 was conducted**

\[
\begin{align*}
x(t) &= ? \\
\text{(h(t))} & \rightarrow \text{Gauge} & y(t) &= (2 - 3e^{-t} + e^{-3t})u(t)
\end{align*}
\]

The Laplace transform of this experiment is

\[
x(t) \ast h(t) = y(t) \iff X(s)H(s) = Y(s)
\]

and we know that \(H(s) = \frac{1}{(s+1)^2}\) and \(Y(s) = \frac{2}{5} - \frac{3}{s+1} + \frac{1}{s+3}\)

according to p. 591 of the textbook. So

\[
X(s) = \frac{Y(s)}{H(s)} = (s+1)^2\left[\frac{2}{5} - \frac{3}{s+1} + \frac{3}{s+3}\right] = \text{algebra} \ldots = \frac{6(s+1)}{s(s+3)}
\]

\[
= \frac{A}{s} + \frac{B}{s+3}
\]

\[A = \frac{6(0+1)}{(0+3)} = 2, \quad B = \frac{6(0+3)}{-3} = -4\]

and \(x(t) = (2 + 4e^{-3t})u(t)\)