A pressure gauge that can be modelled as an LTI system has a time response to a unit step input given by $(1-e^{-t}-te^{-t})u(t)$. For a certain input $x(t)$, the output is observed to be $(2-3e^{-t}+e^{-3t})u(t)$. For this observed measurement, determine the true pressure input to the gauge as a function of time.

Experiment # 1 was conducted

$$\xrightarrow{u(t)} \boxed{\text{Gauge}} \longrightarrow s(t) = (1-e^{-t}-te^{-t})u(t)$$

We know that the step response $s(t)$ is the integral of the impulse response $h(t)$, so...

product rule of derivatives

$$h(t) = \frac{d}{dt}s(t) = (0+e^{-t}+te^{-t}-e^{-t})u(t) + \underbrace{(1-e^{-t}-te^{-t})}_{\text{zero when } t=0}\delta(t)$$

$$\boxed{= te^{-t}u(t)}$$

Experiment # 2 was conducted

$$x(t) = ? \longrightarrow \boxed{\begin{array}{c}\text{Gauge}\\ (h(t))\end{array}} \longrightarrow y(t) = (2-3e^{-t}+e^{-3t})u(t)$$

The Laplace transform of this experiment is

$$x(t) * h(t) = y(t) \longleftrightarrow X(s)H(s) = Y(s)$$

and we know that $H(s) = \frac{1}{(s+1)^2}$ and $Y(s) = \frac{2}{s} - \frac{3}{s+1} + \frac{1}{s+3}$

according to p.591 of the textbook, so

$$X(s) = Y(s)/H(s) = (s+1)^2\left[\frac{2}{s} - \frac{3}{s+1} + \frac{3}{s+3}\right] = \text{algebra} \cdots = \frac{6(s+1)}{s(s+3)}$$

$$= \frac{A}{s} + \frac{B}{s+3} \qquad A = \frac{6(0+1)}{(0+3)} = 2$$

$$B = \frac{6(-3+1)}{-3} = 4$$

and $\boxed{x(t) = (2+4e^{-3t})u(t)}$