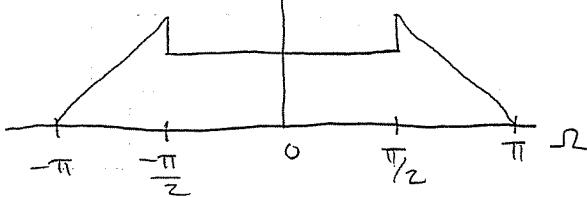
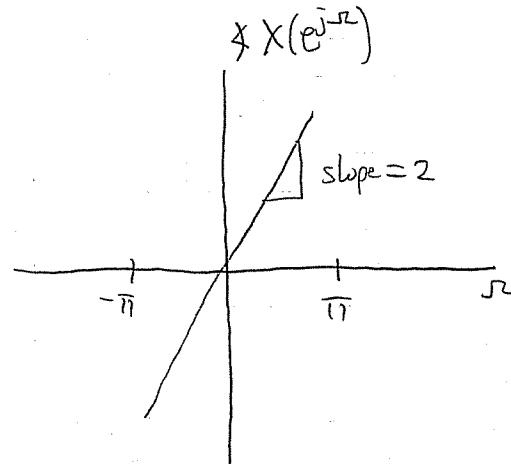


Consider the sequence $x[n]$ whose DTFT is shown below for $-\pi \leq \omega \leq \pi$. We wish to determine whether or not, in the time domain, $x[n]$ is periodic, real, even, and/or of finite energy.

$$|X(e^{j\omega})|$$



$$X(e^{j\omega})$$



- Periodicity in the time domain implies that $x[n]$ has a Fourier Series representation, which would appear in the DTFT as impulses. Since we do not see impulses in the magnitude plot, we conclude that $x[n]$ is not periodic.
- Using the Symmetry properties from OTW Table 5.1, we know that a real-valued $x[n]$ must have a DTFT of even magnitude and a phase function that is odd. This is true of the above plots so $x[n]$ is real.
- If $x[n]$ is even, then, by the symmetry properties for real signals, $X(e^{j\omega})$ must be real and even, but we have
$$X(e^{j\omega}) = |X(e^{j\omega})| e^{j2\omega} \text{ which is complex valued,}$$
 so $x[n]$ is not even.
- Finally since the area under the magnitude curve is finite, and
$$\frac{1}{2\pi} \int_{-2\pi}^{2\pi} |X(e^{j\omega})|^2 d\omega = \sum_{n=-\infty}^{\infty} |x[n]|^2 = \text{finite number, then}$$
 $x[n]$ has finite energy.