Consider an LTI system \( S \) with impulse response
\[
h(t) = \frac{\sin(8\pi(t-1))}{\pi(t-1)}
\]

Determine the response of \( S \) for each of the following excitations:
(a) \( x_1(t) = \cos(12\pi t + \frac{\pi}{4}) \)
(b) \( x_2(t) = \sum_{k=0}^{\infty} \left( \frac{1}{2} \right)^k \sin(6\pi k t) \)
(c) \( x_3(t) = \frac{\sin(8\pi(t+1))}{\pi(t+1)} \)
(d) \( x_4(t) = \left( \frac{\sin(4\pi t)}{\pi t} \right)^2 \)

Using the table of CTFT pairs, and the time shift property and the time scaling property we have
\[
H(f) = e^{j2\pi f} \text{rect}(f/8)
\]
This is an ideal low-pass filter whose passband is in the range \( |f| < 4 \)

(a) The CTFT of \( x_1(t) \) is
\[
X_1(f) = \frac{e^{j\frac{3\pi}{4} f}}{2} \left[ \delta(f-6) + \delta(f+6) \right]
\]
Since none of the signal is in the passband of the filter, the response is
\[
y_1(t) = 0
\]

(b) The CTFT of \( x_2(t) \) is
\[
X_2(f) = \sum_{k=0}^{\infty} \left( \frac{1}{2} \right)^k \left( \frac{1}{2} \right) [\delta(f+3k) - \delta(f-3k)]
\]
The only terms in the summation that are in the passband of the filter are for \( k = 0, 1 \)
\[
X_2(f) = \frac{1}{2} \left[ \delta(f) - \delta(f) \right] + \frac{1}{4} \left[ \delta(f+3) - \delta(f-3) \right]
\]
\[
y_2(t) = \frac{1}{2} \sin(6\pi(t-1))
\]
(c) The CTFT of $x_3(t)$ is $X_3(f) = e^{j2\pi f \cdot \text{rect}(f/8)}$

and so $Y_3(f) = H(f) \cdot X_3(f) = \text{rect}(f/8)$

$\Rightarrow y_3(t) = \frac{\sin(8\pi t)}{\pi t}$

(d) The CTFT of $x_4(t)$ is $X_4(f) = \text{rect}(4f) \ast \text{rect}(4f)$

$= \text{tri}(f/4)$

$Y_4(f) = H(f) X_4(f) = \text{tri}(f/4) \cdot \text{rect}(f/8) e^{j2\pi f t}$

$\Rightarrow y_4(t) = \left[\frac{\sin(4\pi(t-1))}{\pi(t-1)}\right]^2$