Consider an LTI system S with impulse response

$$h(t) = \frac{\sin(8\pi(t-1))}{\pi(t-1)}$$

Determine the response of S for each of the following excitations:

(a) $x_1(t) = \cos\left(12\pi t + \frac{\pi}{2}\right)$

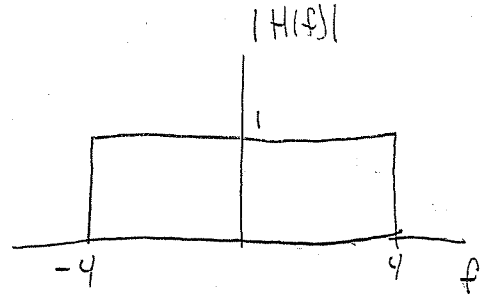
(b) $x_2(t) = \sum_{k=0}^{\infty} \left(\frac{1}{2}\right)^k \sin(6\pi k t)$

(c) $x_3(t) = \frac{\sin(8\pi(t+1))}{\pi(t+1)}$

(d) $x_4(t) = \left(\frac{\sin(4\pi t)}{\pi t}\right)^2$

Using the table of CTFT pairs, and the time shift property and the time scaling property we have

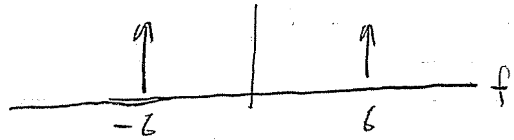
$$H(f) = e^{-j2\pi f}\,\text{rect}(f/8)$$



This is an ideal low-pass filter whose passband is in the range $|f| < 4$

(a) The CTFT of $x_1(t)$ is $X_1(f) = \frac{e^{j\pi/12 f}}{2}\left[\delta(f-6) + \delta(f+6)\right]$

Since none of the signal is in the passband of the filter, the response is



$$\boxed{y_1(t) = 0}$$

(b) The CTFT of $x_2(t)$ is $X_2(f) = \sum_{K=0}^{\infty} \left(\frac{1}{2}\right)^K \left(\frac{j}{2}\right)\left[\delta(f+3K) - \delta(f-3K)\right]$

The only terms in the summation that are in the passband of the filter are for $K = 0, 1$

$$Y_2(f) = \frac{j}{2}\left[\delta(f) - \delta(f)\right] + \frac{j}{4}\left[\delta(f+3) - \delta(f-3)\right]$$

$$\boxed{y_2(t) = \frac{1}{2}\sin(6\pi(t-1))}$$

(c) The CTFT of $x_3(t)$ is $X_3(f) = e^{j2\pi f}\text{rect}(f/8)$

and so $Y_3(f) = H(f)\cdot X_3(f) = \text{rect}(f/8)$

$$\Rightarrow y_3(t) = \frac{\sin(8\pi t)}{\pi t}$$

(d) The CTFT of $x_4(t)$ is $X_4(f) = \text{rect}(4f) * \text{rect}(4f)$

$$= \text{tri}(f/4)$$

$$Y_4(f) = H(f)X_4(f) = \text{tri}(f/4)\,\text{rect}(f/8)\,e^{-j2\pi f}$$

$$\Rightarrow y_4(t) = \left[\frac{\sin(4\pi(t-1))}{\pi(t-1)}\right]^2$$

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