Consider the Fourier transform pair

$$e^{-|t|} \overset{\mathcal{F}}{\longleftrightarrow} \frac{2}{1+\omega^2} \qquad (1)$$

(a) Use the appropriate CTFT properties to find the CTFT of

$$t e^{-|t|} \qquad (2)$$

(b) Use the result from part (a), along with the duality property, to determine the CTFT of

$$\frac{4t}{(1+t^2)^2} \qquad (3)$$

(a) The relationship between (1) and (2) looks like the Differentiation in frequency property in O&W Table 4.1

$$t\,x(t) \overset{\mathcal{F}}{\longleftrightarrow} j\frac{d}{d\omega}X(j\omega)$$

Therefore, $$t e^{-|t|} \longleftrightarrow j\frac{d}{d\omega}\left\{\frac{2}{1+\omega^2}\right\} \boxed{= -\frac{4j\omega}{(1+\omega^2)^2}} \qquad (4)$$

(b) Equation (3) looks identical to (4) except for t and ω

we have (2) and (4) as CTFT pairs

$$x(t) \longleftrightarrow X(j\omega)$$

and 3 can be written as $jX(jt)$

according to the duality property on p. 370

$$X(jt) \longleftrightarrow 2\pi x(-\omega) \qquad \text{therefore}$$

$$j\left(\frac{-4jt}{(1+t^2)^2}\right) \longleftrightarrow j\left(2\pi(-\omega)e^{-|-\omega|}\right)$$

$$\boxed{= -j2\pi\omega e^{-|\omega|}}$$