Given $x(t) \overset{\mathcal{F}}{\longleftrightarrow} X(j\omega)$, express the CTFT of the signals listed below in terms of $X(j\omega)$

(a) $x_1(t) = x(1-t) + x(-1-t)$

(b) $x_2(t) = x(3t-6)$

(c) $x_3(t) = \frac{d^2}{dt^2} x(t-1)$

(a) using the time scaling property we have

$$x(-t) \overset{\mathcal{F}}{\longleftrightarrow} X(-j\omega)$$

using the time shifting property we have

$$x(-t+1) \overset{\mathcal{F}}{\longleftrightarrow} e^{-j\omega t} X(-j\omega) \quad \text{and} \quad x(-t-1) \overset{\mathcal{F}}{\longleftrightarrow} e^{j\omega t} X(-j\omega)$$

Therefore

$$x_1(t) = x(-t+1) + x(-t-1) \overset{\mathcal{F}}{\longleftrightarrow} e^{-j\omega t} X(-j\omega) + e^{j\omega t} X(-j\omega)$$

$$\boxed{= 2X(-j\omega)\cos(\omega)}$$

(b) using the time scaling property we have

$$x(3t) \overset{\mathcal{F}}{\longleftrightarrow} \frac{1}{3} X(j\frac{\omega}{3})$$ and with the time shifting prop

$$x_2(t) = x(3(t-2)) \overset{\mathcal{F}}{\longleftrightarrow} \underline{\underline{e^{-j2\omega} \frac{1}{3} X(j\frac{\omega}{3})}}$$

(c) Using the time differentiation property we have

$$\frac{d}{dt} x(t) \overset{\mathcal{F}}{\longleftrightarrow} j\omega X(j\omega)$$, applying twice we get

$$\frac{d^2}{dt^2} x(t) \overset{\mathcal{F}}{\longleftrightarrow} -\omega^2 X(j\omega)$$

using time shifting we get

$$x_3(t) = \frac{d^2}{dt} x(t-1) \overset{\mathcal{F}}{\longleftrightarrow} \underline{\underline{-\omega^2 X(j\omega) e^{-j\omega t}}}$$