Let $x(t)$ be a periodic signal with fundamental period T_0 and CTFS harmonic function $X[k]$. Derive CTFS harmonic functions of each of the following signals in terms of $X[k]$: [use $f_F = f_0$]

(a) $g(t) = x(t-t_0) + x(t+t_0)$ ← apply Linearity and Time Shifting properties

$$G[k] = e^{-j2\pi k f_F t_0} X[k] + e^{+j2\pi k f_F t_0} X[k] = 2\cos(2\pi k f_F t_0) X[k]$$

(b) $g(t) = Ev\{x(t)\} = \frac{1}{2}\left(x(t) + x(-t)\right)$ ← Apply linearity and time reversal properties

$$G[k] = \frac{1}{2}\left(X[k] + X[-k]\right) = Ev\{X[k]\}$$

(c) $g(t) = Re\{x(t)\} = \frac{1}{2}\left(x(t) + x^*(t)\right)$ ← Apply linearity and conjugation properties

$$G[k] = \frac{1}{2}\left(X[k] + X^*[-k]\right)$$

(d) $g(t) = \dfrac{d^2 x(t)}{dt^2}$ ← apply differentiation property twice

$$G[k] = (j2\pi k f_F)^2 X[k] = -4\pi^2 k^2 f_F^2 X[k]$$

(e) $g(t) = x(3t-1)$ ← apply time scaling and time shifting

$x(3t)$ has fundamental period $T_0/3$ and CTFS harmonic function still given by $X[k]$ (i.e. the only thing that changes is $T_0/3$)

To account for the time shift in $x(3t-1)$ we get

$$G[k] = e^{-j2\pi k(3f_F)(1)} X[k] = e^{-j6\pi k f_F} X[k]$$