Determine whether each of the following statements concerning LTI systems is <u>true</u> or <u>false</u>

(a) If $h(t)$ is the impulse response of an LTI system and $h(t)$ is periodic and nonzero, then the system is unstable.

<u>Answer</u> Since $h(t)$ is periodic and nonzero, $\int_{-\infty}^{\infty} |h(\tau)| d\tau = \infty$

<u>True</u>

(b) The inverse of a causal LTI system is always causal

<u>False</u>, For example, a unit delay $h_1[n] = \delta[n-1]$ ← causal

has an inverse given by $h_2[n] = \delta[n+1]$ ← non-causal

(c) If $|h[n]| \leq K$ for each n, where K is a given number, then the LTI system with $h[n]$ as its impulse response is stable.

<u>False</u> Just because the amplitude of $h[n]$ is bounded, it may still be infinitely long in time, like the ideal accumulator $h[n] = u[n] \Rightarrow \sum_{m=-\infty}^{\infty} |u[n]| = \infty$ ← unstable

(d) If a discrete-time LTI system has an impulse response $h[n]$ of finite duration then the system is stable

If $h[n]$ is bounded then <u>true</u>

If $h[n]$ is unbounded then <u>false</u>

[continued ...]

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(e) If an LTI system is causal, it is stable.

False, consider again the ideal accumulator with $h[n] = u[n]$. This is a causal unstable system

(f) The cascade of a non causal LTI system with a causal one is necessarily noncausal.

False, consider again the unit advance $h_1[n] = \delta[n+1]$ followed by the unit delay $h_2[n] = \delta[n-1]$, the response of this cascaded system is $\delta[n-1] * \delta[n+1] = \delta[n] \leftarrow$ causal (memoryless)

(g) A continuous-time LTI system is stable if and only if its step response is absolutely integrable — that is, if and only if

$$\int_{-\infty}^{\infty} |h_{-1}(\tau)| d\tau < \infty$$

False For example $h(t) = e^{-t}u(t)$, we have $\int_{-\infty}^{\infty} |e^{-\tau}u(\tau)| d\tau = 1$ so the system is stable, but

$$h_{-1}(t) = \int_{-\infty}^{t} e^{-\tau} u(\tau) d\tau = (1 - e^{-t}) u(t)$$

and $\int_{-\infty}^{\infty} |(1-e^{-\tau})u(\tau)| d\tau = \infty$, Thus the system is stable but the step response is not absolutely integrable

(h) A discrete-time LTI system is causal if and only if its step response is zero for $n < 0$

True $\delta[n] \rightarrow h[n]$ and $u[n] \rightarrow h_{-1}[n]$

we can write $u[n]$ as $u[n] = \sum_{k=0}^{\infty} \delta[n-k]$, therefore

$h_{-1}[n] = \sum_{k=0}^{\infty} h[n-k]$, if $h_{-1}[n] = 0$ for $n<0$ then $h[n] = 0$ for $n<0$ and the system is causal.