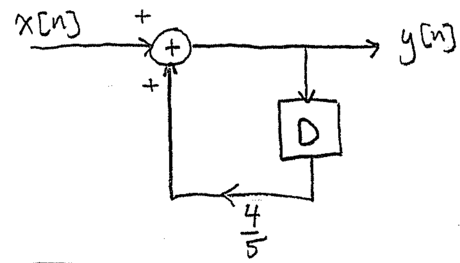
Classify the discrete-time system given by

$y[n] = x[n] + \frac{4}{5} y[n-1]$



Homogeneity

Let $x_1[n] = g[n]$, then $y_1[n] - \frac{4}{5} y_1[n-1] = g[n]$ (1)

now Let $x_2[n] = K g[n]$, then $y_2[n] - \frac{4}{5} y_2[n-1] = K g[n]$

multiply (1) by K $\rightarrow$ $K y_1[n] - \frac{4}{5} K y_1[n-1] = K g[n]$

therefore $K y_1[n] - \frac{4}{5} K y_1[n-1] = y_2[n] - \frac{4}{5} y_2[n-1]$

this can only be true if $y_2[n] = K y_1[n]$ $\Rightarrow$ homogeneous

Additivity

Let $x_1[n] = g[n]$, then $y_1[n] - \frac{4}{5} y_1[n-1] = g[n]$ (1)

Let $x_2[n] = h[n]$, then $y_2[n] - \frac{4}{5} y_2[n-1] = h[n]$ (2)

Let $x_3[n] = g[n] + h[n]$ then $y_3[n] - \frac{4}{5} y_3[n] = g[n] + h[n]$

adding (1) and (2) gives us

$y_1[n] + y_2[n] - \frac{4}{5}\left(y_1[n-1] + y_2[n-1]\right) = y_3[n] - \frac{4}{5} y_3[n-1]$

- this can only be true if $y_3[n] = y_1[n] + y_2[n]$ $\Rightarrow$ additive

the system is then Linear

Time invariance

Let $x_1[n] = g[n]$, then $y_1[n] - \frac{4}{5} y_1[n-1] = g[n]$ (1)

Let $x_2[n] = h[n-n_0]$ then $y_2[n] - \frac{4}{5} y_2[n-1] = g[n-n_0]$ (2)

re-write (1) as $y_1[n-n_0] - \frac{4}{5} y_1[n-n_0-1] = g[n-n_0]$ (3)

equating (2) and (3) yields

$y_2[n] - \frac{4}{5} y_2[n-1] = y_1[n-n_0] - \frac{4}{5} y_1[n-n_0-1]$ which can only be true if $y_2[n] = y_1[n-n_0]$

$\Rightarrow$ time-invariant

Stability

The homogeneous solution to this equation is

$$y[n] = K_h (4/5)^n$$ ← the zero excitation response goes to zero as $n \to \infty$

the response to $x[n] = \delta[n]$ goes to zero as $n \to \infty$

the response to $x[n] = u[n]$ goes to 5 as $n \to \infty$

$\Rightarrow$ stable

Causality

consider the response to $x[n] = \delta[n]$ of the system initially at rest

$$y[n] = 0, \quad n < 0$$

$$y[0] = \delta[0] = 1$$

$$y[n] = \left(\tfrac{4}{5}\right)^n, \quad n \geq 0$$

the system does not respond before $x[n]$ is applied

$\Rightarrow$ causal

Memory

the system responds to $x[n] = \delta[n]$ for <u>all</u> $n > 0$

$\Rightarrow$ the system has memory

Invertability

Based on the system equation $y[n] - \frac{4}{5} y[n-1] = x[n]$

If you have the current output sample $y[n]$ and the previous output sample $y[n-1]$ then

$$y[n] - \frac{4}{5} y[n] = \text{the current input sample} = x[n]$$

so $\Rightarrow$ invertable