(a) Show that causality for a continuous-time linear system is equivalent to the following statement:

"For any time t_0 and any excitation $x(t)$ such that $x(t) = 0$ for $t < t_0$, the corresponding response $y(t)$ must also be zero for $t < t_0$."

We have the following statements

$$x(t) = 0 \text{ for } t < t_0 \qquad (1)$$

$$y(t) = 0 \text{ for } t < t_0 \qquad (2)$$

$$\text{the system is causal} \qquad (3)$$

We will first prove that if (1) and (2) are true, then (3) is true, Second, we will prove that if (3) is true, then (1) means that (2) is true.

Consider the signals $x_1(t)$ and $x_2(t)$, which are identical for $t < t_0$, but for $t > t_0$, $x_1(t) \neq x_2(t)$. The system is linear, so

$$x_1(t) - x_2(t) \longrightarrow y_1(t) - y_2(t)$$

Since $x_1(t) - x_2(t) = 0$ for $t < t_0$ [statement (1)] and we also assume $y_1(t) - y_2(t) = 0$ for $t < t_0$ [statement (2)], this implies that $y_1(t) = y_2(t)$ for $t < t_0$. In otherwords, the response is not affected by future values of the excitation. => Causal (3)

Assuming the system is causal (3), and we have $X(t) = x_1(t) - x_2(t)$ and $X(t) = 0$ for $t < t_0$ as outlined above [statement (1)]. The response is $y(t) = y_1(t) - y_2(t)$, Now, since the system is causal, $x_1(t) = x_2(t)$ for $t < t_0$ implies that $y_1(t) = y_2(t)$ for $t < t_0$, Therefore, $y(t) = 0$ for $t < t_0$ [statement (2)]

An analogous statement can be made for a discrete-time system

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(b) Find a <u>nonlinear system</u> that satisfies the foregoing statement but is not causal

Consider the system $y(t) = x(t)\,x(t+1)$.

when $x(t) = 0$ for $t < t_0$, the response will be $y(t) = 0$ for $t < t_0$. This system is nonlinear and noncausal

(c) Find a <u>nonlinear system</u> that is causal but does not satisfy the foregoing statement.

Consider the system $y(t) = x(t) + 1$. This system is nonlinear and causal, and does not satisfy the statement in part (a).