For each of the following input-output relationships, determine whether the corresponding system is linear, time invariant, or both.

(a) \( y(t) = t^2 x(t+1) \)

Linear:
\[
x_1(t) = g(t) \rightarrow y_1(t) = t^2 g(t+1), \quad x_2(t) = h(t) \rightarrow y_2(t) = t^2 h(t+1)
\]

Nonlinear:
\[
\alpha x_1(t) + \beta x_2(t) \rightarrow t^2 \left( \alpha g(t+1) + \beta h(t+1) \right) \neq \alpha y_1(t) + (\beta y_2(t)
\]

Time-invariant:
\( y(t) = t^2 x(t+1) \)

Time-varying:

The system needs to know what time it is to compute its output.

(b) \( y[n] = x^2[n-2] \)

Linear:
\[
\alpha g[n] + \beta h[n] \rightarrow (\alpha g[n-2] + \beta h[n-2])^2
\]

Nonlinear:

Time-invariant:
\( x_1[n] = g[n] \rightarrow y_1[n] = g^2[n-2] \)

Time-varying:
\( x_2[n] = g[n-n_0] \rightarrow y_2[n] = g^2[n-n_0-2] \neq y_1[n-n_0] \)

(c) \( y[n] = x[n+1] - x[n-1] \)

\[
\alpha g[n] + \beta h[n] \rightarrow \alpha (g[n+1] - g[n-1]) + \beta (h[n+1] - h[n-1])
\]

Time-invariant:
\( x[n] = g[n-n_0] \rightarrow g[n-n_0+1] - g[n-n_0-1] = y[n-n_0] \)

(d) \( y(t) = \text{od} \{ x(t) \} = \frac{x(t) - x(-t)}{2} \)

Linear:
\[
\alpha g(t) + \beta h(t) \rightarrow \frac{\alpha g(t) - \alpha g(-t)}{2} + \beta \frac{h(t) - h(-t)}{2}
\]

Time-invariant:
\( x_1(t) = g(t) \rightarrow y_1(t) = \frac{g(t) - g(-t)}{2} \)

Time-varying:
\( x_2(t) = g(t-t_0) \rightarrow y_2(t) = \frac{g(t-t_0) - g(-t-t_0)}{2} \neq y_1(t-t_0) \)