Let \( x[n] \) be a discrete-time signal and let

\[
y_1[n] = x[2n] \quad \text{and} \quad y_2[n] = \begin{cases} x[n/2], & \text{if } n \text{ even} \\ 0, & \text{if } n \text{ odd} \end{cases}
\]

\( y_1[n] \) is a speeded up version of \( x[n] \), and \( y_2[n] \) is a slowed down version of \( x[n] \).

Consider the following statements:

1. If \( x[n] \) is periodic, then \( y_1[n] \) is periodic.
2. If \( y_1[n] \) is periodic, then \( x[n] \) is periodic.
3. If \( x[n] \) is periodic, then \( y_2[n] \) is periodic.
4. If \( y_2[n] \) is periodic, then \( x[n] \) is periodic.

Determine whether each of these is true, if so determine the relationship between the fundamental periods of the two signals. If false, produce a counterexample to the statement.

1. True: \( x[n] = x[n+N] \); \( y_1[n] = y_1[n+N_0] \), i.e. \( y_1[n] \) is periodic with period \( N_0 = N/2 \) if \( N \) is even, and \( N_0 = N \) if \( N \) is odd.

2. False: \( y_1[n] \) has less information than \( x[n] \), and we don't know what was in the samples of \( x[n] \) that are not present in \( y_1[n] \).

Let \( x[n] = g[n] + h[n] \), \( g[n] = \begin{cases} 1, & \text{if } n \text{ even} \\ 0, & \text{if } n \text{ odd} \end{cases} \) periodic,

\[
h[n] = \begin{cases} 0, & \text{if } n \text{ even} \\ 1, & \text{if } n \text{ odd} \end{cases} \text{aperiodic}
\]

3. True: \( x[n] = x[n+N] \); \( y_2[n] = y_2[n+N_0] \), where \( N_0 = 2N \).

4. True: \( y_2[n] = y_2[n+N] \); \( x[n] = x[n+N_0] \), where \( N_0 = N/2 \).