Lab 7. Approximation of the Continuous Time Fourier Transform

The continuous time Fourier Transform can be approximated by the following sum:

\[ \int_{-\infty}^{\infty} x(t) e^{-j2\pi f t} dt = \lim_{\tau \to 0} \sum_{m=-\infty}^{\infty} x(m\tau) e^{-j2\pi f m\tau} \]

Given that you have a record of \(T\) seconds sampled every \(\tau\) seconds resulting in a total of \(N\) samples (Note: \(T = N\tau\) of the signal \(x(t)\) then the continuous time Fourier Transform can be approximated by:

\[ \int_{-\infty}^{\infty} x(t) e^{-j2\pi f t} dt = \int_{0}^{T} x(t) e^{-j2\pi f t} dt \approx \sum_{m=0}^{N-1} x(m\tau) e^{-j2\pi f m\tau} \]

Let

\[ f_0 = \frac{1}{T} = \frac{1}{N\tau} \]

so

\[ f = nf_0 = \frac{n}{N\tau} \]

and define

\[ X_n = \sum_{m=0}^{N-1} x(m\tau) e^{-j2\pi fm\tau} = \tau \sum_{m=0}^{N-1} x(m\tau) e^{-j2\pi m/n} \]

Now \(X_n\) can be viewed as an approximation for the continuous time Fourier Transform of \(x(t)\) at \(f = nf_0\). Note that \(X_n\) is a complex number.

a). Given \(x(t)\), \(T\), \(\tau\), and write a Matlab routine to find \(X_n\).

b). Test and validate your routine for \(x(t) = u(t)e^{-t}\) with \(\tau = 0.01\) sec and \(T = 1.28\) sec by analytically determining the Fourier Transform for \(x(t) = u(t)e^{-t}\) and graph \(|X(f)|\) and \(|X_n|\) on the same plot, also graph the phase of \(X(f)\) and \(X_n\) on the same plot.

c). Repeat part b) for \(T = 5.12\) sec.

d). Repeat part b) for \(\tau = 0.1\) sec and \(T = 12.8\) sec.

e). Comment on the accuracy of the approximation as \(T\) increases.