EECS 461 Expectation Homework Problems
Probability and Statistics
Due Date: TBD, 2008

Name: ______________________________

1. Let X be a random variable with mean μ_x and variance σ_x^2. Show that the random variable $Y = X + b$ has mean $\mu_y = \mu_x + b$ and variance $\sigma_y^2 = \sigma_x^2$. This problem demonstrates that **any random variable** can be thought of as a **zero mean random variable plus a constant**; the addition of the constant changes the mean but not the variance.

2. Let X be a random variable with mean μ_x and variance σ_x^2. Show that the random variable $Y = aX$ has mean $\mu_y = a\mu_x$ and variance $\sigma_y^2 = a^2\sigma_x^2$.

3. Let X be a random variable with mean μ_x and variance σ_x^2. Suppose we want to obtain a version of X that has **zero mean** and **unit variance**. In other words, we want to obtain the random variable $Z = h(X)$ such that Z has zero mean and unit variance. Based on the results from the first two problems, show the exact relationship between X and Z.

4. Let W and X be random variables with respective means μ_w and μ_x and variances σ_w^2 and σ_x^2. Show that the random variable $Y = W + X$ has mean $\mu_y = \mu_w + \mu_x$.

5. Let W and X be **independent** random variables with **zero mean** and respective variances σ_w^2 and σ_x^2. Show that the random variable $Y = W + X$ has variance $\sigma_y^2 = \sigma_w^2 + \sigma_x^2$. It is reasonable to ask: What if W and X have non-zero means? Does σ_y^2 change? Based on the result of Problem 1, argue that σ_y^2 does not change when we add a non-zero mean to W and X.

6. Let $\{X_i\}$ be a set of n independent random variables, each with an individual mean μ and variance σ^2. Based on all of the above, show that the *sample mean*

$$\overline{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$$

has mean $\mu_{\overline{X}} = \mu$ and variance $\sigma_{\overline{X}}^2 = \sigma^2/n$.