Simulation of Uncoded Modulations

EECS 869: Error Control Coding
Fall 2013

Complete the following tasks and summarize your findings, observations, and conclusions in a report. Include your source code as an appendix to your report.

1. **Generate a BER Plot for Uncoded BPSK using a Discrete Model.** Technically speaking, BPSK is a *bandpass* transmission centered around some RF carrier frequency; because we are doing a *baseband* transmission, this should really be called *binary PAM*. This model is

$$r_n = a_n + w_n \tag{1}$$

where $a_n \in \{\pm A\}$ is the transmitted information symbol at time step n and w_n is a Gaussian random variable with zero mean and variance $N_0/2$. Viewed over time, $\{a_n\}$ is a discrete-time random process with independent and uniformly distributed (i.u.d.) values and $\{w_n\}$ is a discrete-time additive white Gaussian noise (AWGN) process with zero mean and power spectral density (PSD) $N_0/2$. All of the quantities in (1) are entirely real valued. We refer to (1) as a *discrete model* for BPSK because it is in discrete time and there is *one* received sample per symbol time (i.e., the received signal consists of "symbol-spaced" samples). As defined above, $\{a_n\}$ are drawn from an *antipodal* alphabet, which means the two possible values have the same magnitude A but opposite signs. Typically, information bits arrive at the transmitter in *logical* form, meaning 0's and 1's. With BPSK, there are two possible mappings from logical bits to antipodal symbols and both yield identical performance. Your task is to generate a bit error rate (BER) plot for the BPSK model in (1) with the following requirements:

   - Run your simulation for values of $E_b/N_0 = 0, 1, 2, \cdots, 10$ dB (E_b is the energy per information bit and is related to A).
   - Run each simulation point until at least 1,000,000 bits are transmitted *and* at least 100 bit errors are counted.

As way of double-checking the accuracy of your simulation, the theoretical bit error probability of BPSK is given by

$$P_b = Q\left(\sqrt{2\frac{E_b}{N_0}}\right) \tag{2}$$

where $Q(x) = \int_x^\infty e^{u^2/2}\, du$. This function is plotted in Figure 1. When you plot your BER points, they should lie on this curve. Also, as a matter of style, when one plots data points from a *simulation*, it is customary to explicitly show the actual data points with some sort of markers (circles, asterisks, triangles, etc.). Notice in Figure 1 there are no such markers. This is acceptable because (2) describes a curve that is valid over continuous values of E_b/N_0. This is not true of simulation points, which are technically valid only for the discrete values of E_b/N_0 that were simulated, hence the markers.

2. **Generate a BER Plot for Uncoded BPSK using a Waveform Model.** The discrete model in (1) is nice because of its simplicity, but it does not take into account some important details. A more realistic model is the *waveform model* for the received signal, which is

$$r(t) = \sum_i a_i p(t - iT_s) + w(t) \tag{3}$$

where $p(t)$ is a unit-energy pulse shape, T_s is the symbol duration, and $w(t)$ is a continuous-time AWGN process with zero mean and PSD $N_0/2$. Some common pulse shapes are non-return-to-zero (NRZ) and square-root spectral raised-cosine (SRRC). In this class, we will never have access to a continuous-time signal. The best we can do is sample (3) at N samples per symbol interval, where $N = T_s/T$, i.e., the samples are spaced by T seconds. With proper processing (i.e., matched filtering, downsampling to the symbol rate at the proper sampling instant, etc.) the waveform model converges with the discrete model. Thus, the discrete model is often used for simplicity, but one must never forget the many important details and assumptions that must be observed when linking the two models. Your task is to generate a BER plot for the BPSK model in (3) with the following requirements:

- Run your simulation for a NRZ pulse shape using a value of $N = 8$ (unless you feel like using a different value)
- Run your simulation for values of $E_b/N_0 = 0, 1, 2, \cdots, 10$ dB.
- Run each simulation point until at least 1,000,000 bits are transmitted *and* at least 100 bit errors are counted.

The theoretical value of P_b for BPSK does not change for the waveform model.

3. **Generate BER and SER Plots for Uncoded QPSK using a Discrete Model.** The received signal model for *quadrature* phase-shift keying (QPSK) is

$$r_n = a_n + w_n \tag{4}$$

where $a_n \in \{A(\pm 1 \pm j)\}$ and $j = \sqrt{-1}$. In this instance, $\{w_n\}$ is complex-valued AWGN where the real and imaginary parts are independent of one another and each has zero mean and PSD $N_0/2$. Each transmission of a_n carries *two* information bits. There are a number of possible mappings between pairs of logical bits and transmitted symbols, however the best performance is achieved when a *Gray code* is used. Your task is to generate a bit error rate (BER) plot and a symbol error rate (SER) plot for the QPSK model in (4) with the following requirements:

- Run your simulation for values of $E_b/N_0 = 0, 1, 2, \cdots, 10$ dB (remember that E_b is the energy per information bit and is different from the energy per symbol, or E_s).
- Run each simulation point until at least 1,000,000 bits are transmitted *and* at least 100 bit errors are counted.

The theoretical value of P_b for QPSK is the same as with BPSK as long as Gray coding is used. The probability of symbol error, which is usually referred to simply as the *probability of error*, is given by

$$P(E) = 2Q\left(\sqrt{2\frac{E_b}{N_0}}\right). \tag{5}$$

This curve is also plotted in Figure 1.

4. **Generate PSD Plots for BPSK and QPSK Waveforms.** Based on the discrete model for QPSK in (4), you can probably infer the waveform model for QPSK. Here are a couple of additional concepts to consider. The symbol rate is $R_s = 1/T_s$. The bit rate is $R_b = 1/T_b$, where T_b is the bit duration. It should be obvious that $T_s = T_b$ for BPSK and $T_s = 2T_b$ for QPSK, with the obvious ramifications on the respective symbol rates. An important property for any transmitted signal is its PSD. The PSD is the average spectrum, or "expected value" of the spectrum. This can be derived analytically. However, we

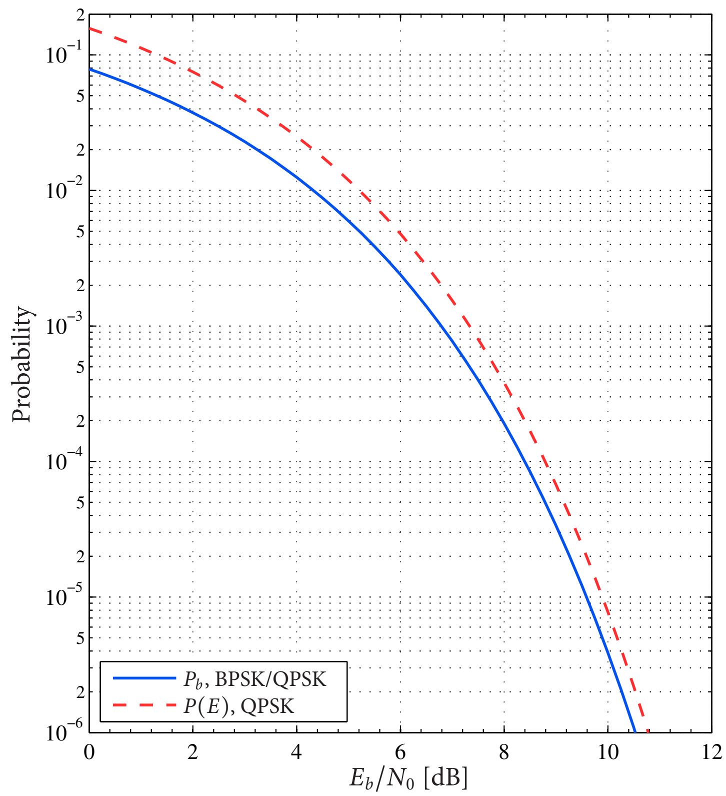

Figure 1: Theoretical values for the probability of bit error, P_b, and the probability of symbol error, $P(E)$, for BPSK and QPSK.

will adopt a "simulation-based" approach here. If the transmitted signal (without noise) is $s(t)$ and its Fourier transform is $S(f)$, then the PSD is

$$P(f) = \lim_{K\to\infty} \frac{1}{K} \sum_{k=0}^{K-1} |S_k(f)|^2 \tag{6}$$

where K "trial" values of the transmitted signal are generated, each with a different set of random information bits. The signal generated in the k-th trial is $s_k(t)$ and its Fourier transform is $S_k(f)$. Your task is to generate PSD plots for BPSK and QPSK waveforms with the following requirements:

- Recall that we do not have the continuous-time waveform, but only the discrete-time waveform sampled at N samples per symbol interval. Thus, we will be using the FFT to approximate the Fourier transform. You should label the abscissa (x-axis) of your plots with frequency normalized to the bit rate, i.e. f/R_b. Pick a value of K that yields plots that are nice and smooth. You also need to pick the number of information bits in each trial signal.
- Run your simulation for a NRZ pulse shape using a value of N that gives you results that you are happy with.
- Try out the MATLAB function PWELCH and see if you are able to get similar results.

Hint: the PSD of QPSK is twice as compact as the PSD of BPSK when plotted against f/R_b.