EECS 312 – Electronic Circuits I –Notes
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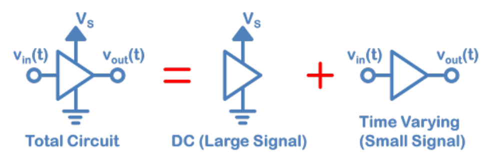
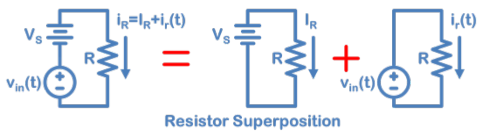
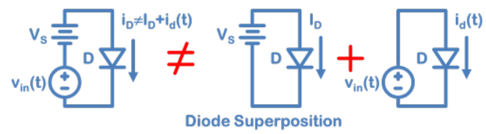
The Diode Small Signal Model

With many circuits, there are two stages of analysis.
1. DC or constant source analysis to determine the operating point of the circuit: DC analysis
2. Time-varying analysis, which relates to the transmission of information: Transfer Functions



For example, an amplifier is initially analyzed using constant or DC source voltage, V_S. Next, the amplifier is analyzed when an input signal is applied to determine the output signal, or amplifier gain $v_{out}(v_{in})$. The figures to the left and below shows this by separating the circuit into two, one with V_S and the other with $v_{in}(t)$. The separation of sources (V_S and $v_{in}(t)$) into two circuits is perfectly fine and can be justified using superposition, as long as all the components used in the amplifier are linear. The diode is not a linear device, and as a result, superposition does not work (shown below with syntax explanation).





$$i_R = \frac{V_S + v_{in}(t)}{R} = \frac{V_S}{R} + \frac{v_{in}(t)}{R} = I_R + i_r(t) \qquad i_D = I_S e^{\frac{V_S + v_{in}(t)}{nV_T}} \neq I_S e^{\frac{V_S}{nV_T}} + I_S e^{\frac{v_{in}(t)}{nV_T}}$$

The small signal model (approximation) is used to linearize the diode for superposition.

Syntax: $\text{lowercase}_{\text{uppercase}}$ is the total value (sum of large signal and small signal values).
$\text{uppercase}_{\text{uppercase}}$ is the large signal DC value.
$\text{lowercase}_{\text{lowercase}}$ is the small signal time-varying value.

Requirement: The small-signal time-varying sources are much less than nV_T (or $v_d(t)/nV_T$ is small).

$$v_D = V_D + v_d(t) \text{ and } |v_d(t)| \ll |nV_T|$$

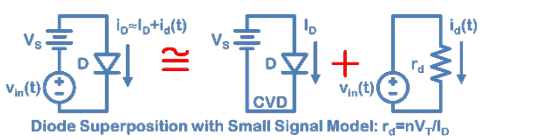
Approximation: Expand the diode exponential equation using a Taylor series at the DC bias point (V_D).

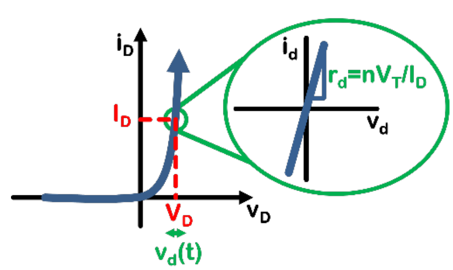
$$i_D(v_D) = i_D(V_D) + i_D{}'(V_D)(v_D - V_D) + \frac{1}{2} i_D{}''(V_D)(v_D - V_D)^2 + \frac{1}{6} i_D{}'''(V_D)(v_D - V_D)^3 + \cdots$$

$$i_D(v_D) = I_S e^{\frac{V_D}{nV_T}} + \frac{I_S}{nV_T} e^{\frac{V_D}{nV_T}} v_d(t) + \frac{I_S}{2(nV_T)^2} e^{\frac{V_D}{nV_T}} v_d^2(t) + \frac{I_S}{6(nV_T)^3} e^{\frac{V_D}{nV_T}} v_d^3(t) + \cdots$$

Since $|v_d(t)|$ is small then $|v_d(t)|^2$ and $|v_d(t)|^3$ and so on are really small and can be ignored. The first term is the diode exponential equation using the large-signal DC source (analyzed using CVD model) and the second term of the expansion is a linear relationship and can be modeled using a resistor, $r_d = nV_T/I_D$.

Model:





$$i_D \cong I_S e^{\frac{V_D}{nV_T}} + \frac{I_S e^{\frac{V_D}{nV_T}}}{nV_T} v_d(t) = I_S e^{\frac{V_D}{nV_T}} + \frac{I_D}{nV_T} v_d(t)$$

$$i_D \cong I_S e^{\frac{V_D}{nV_T}} + \frac{v_d(t)}{r_d}$$